CHAPTER 3

Right Triangle Trigonometry

Suppose you need to calculate the distance across a river for the construction of a bridge or monument. Each of these distances can be calculated using the properties of right triangles, similar triangles and trigonometry. Trigonometry is the branch of Mathematics that studies the relationships between angles and the lines that form them in triangles. It was first developed for use in astronomy and geography. Today, trigonometry is used in surveying, navigation, engineering, construction, and the sciences to explore the relationships between the side lengths and angles of triangles.

HOME

Lesson 1 - The Tangent Ratio

Measurement

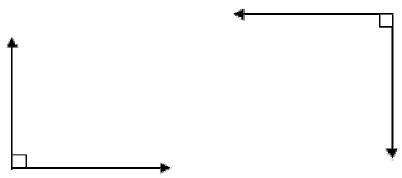
4. Develop and apply the primary trigonometric ratios (sine, cosine, tangent) to solve problems that involve right triangles.

Lesson 1: The Tangent Ratio

Chapter 3- Lesson 1 The Tangent Ratio
Review of Terminology

Definition: A **right angle** is an angle that measures exactly 90°.

Example:



Usually, right angles are designated by the little square as shown in the two examples

Definition: A **straight angle** is an angle that measures exactly 180°.

Example:

Essentially, straight angles are another name for a straight line.

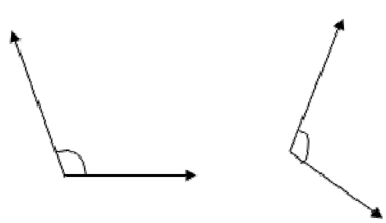
Terminology 1

Definition: An **acute angle** is an angle that measures less than 90°.

Example:

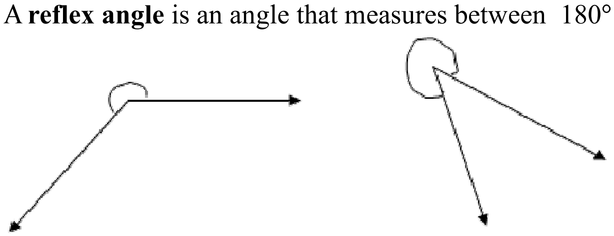
Definition: An **obtuse angle** is an angle that measures greater than 90° but less than 180°.

Example:



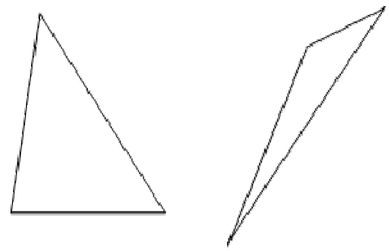
Definition: A **reflex angle** is an angle that measures between 180° and 360°

Example:

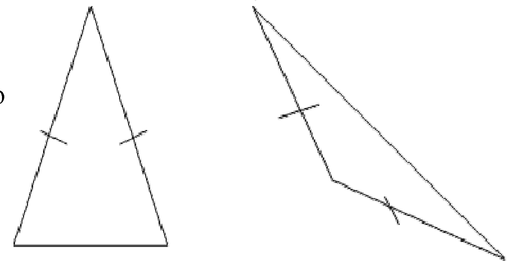


Terminology 2

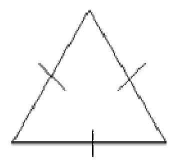
A triangle is said to be **scalene** if all three of the sides have different length (similarly, all three of the angles within the triangle will be different)



A triangle is said to be **isosceles** if two of the three sides are equal in length (similarly, two of the three angles within the triangle will have the same measure):

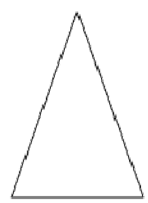


A triangle is said to be **equilateral** if all three sides have the same length (similarly, all three of the angles within the triangle will have the same measure):

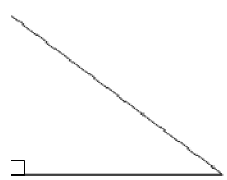


Terminology 3

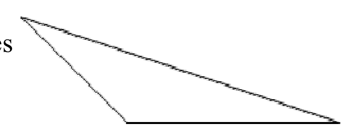
A triangle is said to be **acute** if all three of the angles inside the triangle are acute:



<u>**A triangle is said to be**</u> **right** if one of the angles inside the triangles is a right angle (90°):



<u>**A triangle is said to be**</u> **obtuse** if one of the angles inside the triangle is obtuse:

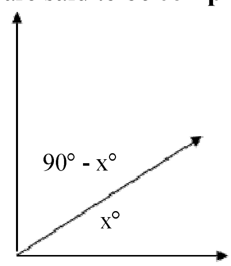


Terminology 4

Angle Relationships

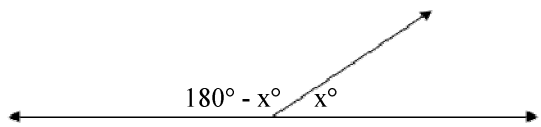
There are certain relationships that always exist with angles and triangles that you should become familiar with:

Definition: Two angles are said to be **complementary angles** if their sums add up to 90°:



Two angles are said to be **supplementary angles** if their sums add up to 180°:

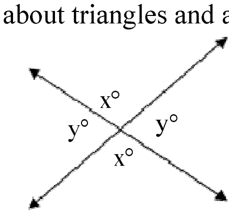
Example:



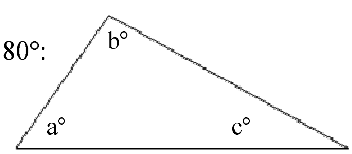
Terminology 5

Other important things to remember about triangles and angles:

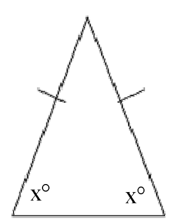
Vertically opposite angles are equal:



The sum of the angles of any triangle is always 180°:

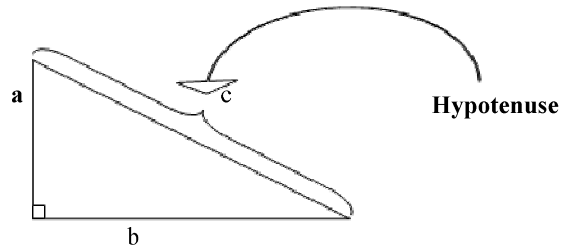


In any isosceles triangle, the base angles are equal:



Terminology 6

Review of Pythagorean Theorem



The formula for the Pythagorean Theorem is:

$$a^2 + b^2 = c^2$$

Review of the Pythagorean Theorem

Similar Triangles Activity

Work with a partner using the grid below, a ruler and a protractor.

A. On the grid, draw a right ΔABC with angle h = 90°.

B. Each of you draws a different right triangle that is **similar**. **(**Recall that two triangles are similar if one triangle is an enlargement or a reduction of the other. Also, all the angles will all be the same.)

C. Measure the side and angles of each triangle. Label your diagrams with the measures.

D. The two shorter sides of a right triangle are its legs. Calculate the ratio $\frac{BC}{BA}$ of the legs as a decimal.

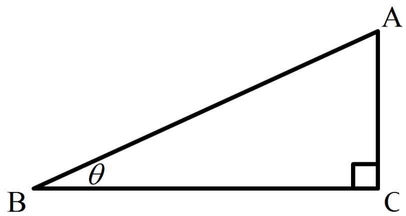
E. Calculate the corresponding ratio for each of the similar triangles.

F. How do the ratios compare?

G. What do you think the value of each ratio depends on?

Similar Triangles Activity

The **tangent ratio** is defined as the *ratio of the length of the side* ***opposite*** *to angle θ and the length of the side* ***adjacent*** *to angle θ.*



$$\tan\theta = \frac{\text{length of AC}}{\text{length of BC}}$$

Tangent Ratio Notes

Example 1 Determining the Tangent Ratio Given the Angle

State the tangent of $38°$ to 4 decimal places. Explain what this means.

Example 1

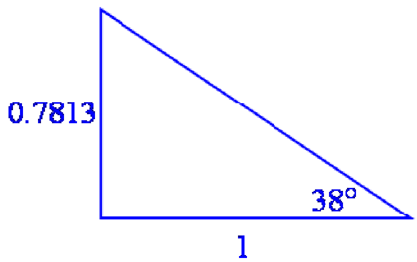
Example 1 Determining the Tangent Ratio Given the Angle

State the tangent of 38° to 4 decimal places. Explain what this means.

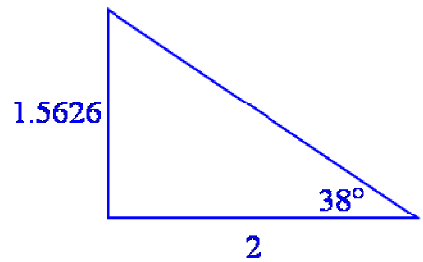
$$\tan 38° \simeq 0.7813$$

This means that from the angle 38°, the opposite side is 0.7813 times the size of the adjacent side.

IE:



OR



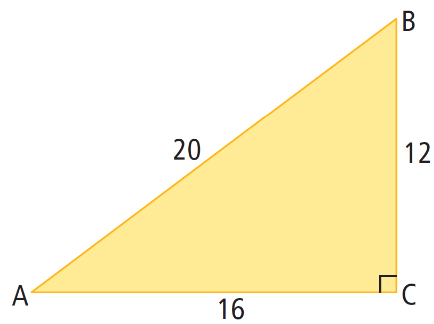
Example 1 Solution

Example 2 Writing the Tangent Ratio

Determine the simplified ratios:

a) tan A

b) tan B



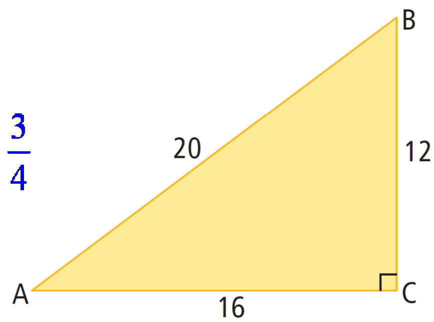
Example 2

Example 2 Writing the Tangent Ratio

Determine the simplified ratios:

a) tan A

$$\tan A = \frac{\textit{side opposite to } \angle A}{\textit{side adjacent to } \angle A} = \frac{12}{16} = \frac{3}{4}$$



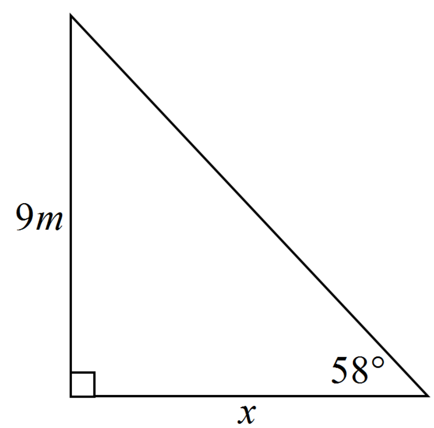
b) tan B

$$\tan B = \frac{\textit{side opposite to } \angle B}{\textit{side adjacent to } \angle B} = \frac{16}{12} = \frac{4}{3}$$

Example 2 Solution

Example 3 Using the Tangent Ratio to Solve for a Side

Solve for the unknown value, correct to the nearest hundredth.



Step One:	Label
Step Two:	Formula
Step Three:	Substitute
Step Three:	Cross-multiply

Example 3

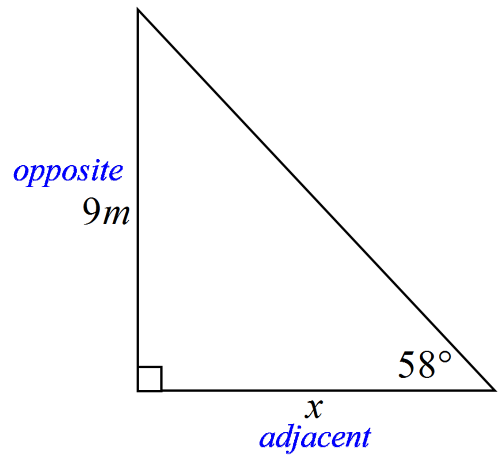
Example 3 Using the Tangent Ratio to Solve for a Side

Solve for the unknown value, correct to the nearest hundredth.

$$\tan 58° = \frac{\textit{side opposite to } 58°}{\textit{side adjacent to } 58°}$$

$$\frac{\tan 58°}{1} = \frac{9}{x}$$

$$x = 5.62$$

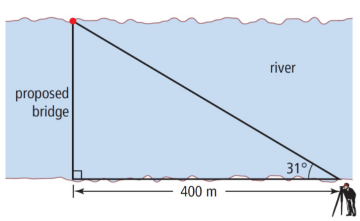


Step One: Label
Step Two: Formula
Step Three: Substitute
Step Three: Cross-multiply

Example 3 Solution

Your Turn

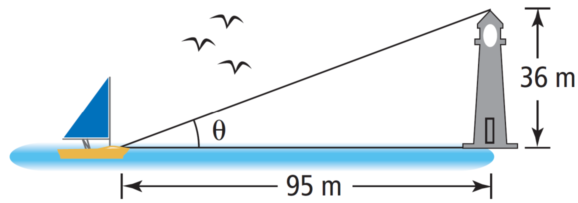
A surveyor wants to determine the width of a river for a proposed bridge. The distance from the surveyor to the proposed bridge site is 400 m. The surveyor uses a theodolite to measure angles. The surveyor measures a 31° angle to the bridge site across the river. Determine the width of the river, to the nearest metre.



Your Turn 3

Example 4

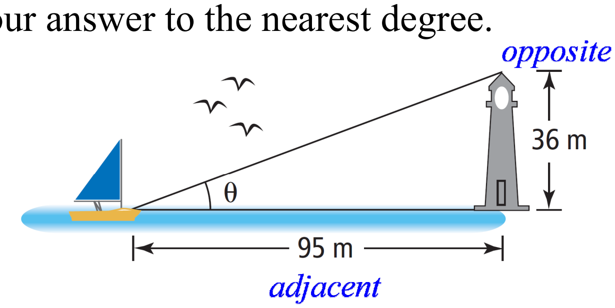
A small boat is 95 m from the base of a lighthouse that has a height of 36 m above sea level. Determine the angle from the boat to the top of the lighthouse. Express your answer to the nearest degree.



Example 4

Example 4

A small boat is 95 m from the base of a lighthouse that has a height of 36 m above sea level. Determine the angle from the boat to the top of the lighthouse. Express your answer to the nearest degree.



$$\tan\theta = \frac{\textit{side opposite to } \theta}{\textit{side adjacent to } \theta}$$

$$\tan\theta = \frac{36}{95}$$

$$\theta = \tan^{-1}\left(\frac{36}{95}\right)$$

$$\theta \simeq 21°$$

The angle is about 21°.

Example 4 Solution

Your Turn

A radio transmission tower is to be supported by a guy wire. The wire reaches 30 m up the tower and is attached to the ground a horizontal distance of 14 m from the base of the tower. Determine the angle of elevation the guy wire forms with the ground, to the nearest degree.

Your Turn 4

End of Lesson

Assignment: Page 107 #1, 3a-c, 4, 6, 7, 9, 13
Challenge: Page 107 # 16

Assignment 1

Lesson 2 - The Sine and Cosine Ratios

Measurement

4. Develop and apply the primary trigonometric ratios (sine, cosine, tangent) to solve problems that involve right triangles.

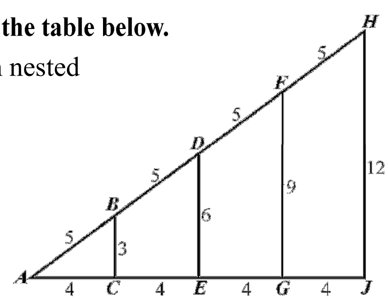
Lesson 2: The Sine and Cosine Ratios

Nested Triangles Activity

Work with a partner to complete the table below.

How are the acute angles in each nested triangle related? Explain.

How are the triangles related?



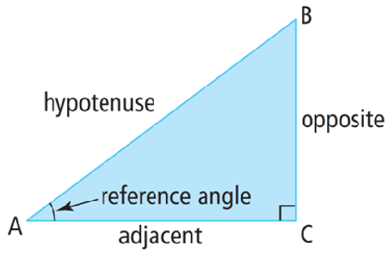
Triangle	Measures of Sides			Ratios	
	Hypotenuse	Side opposite ∠A	Side adjacent to ∠A	Side opposite ∠A / Hypotenuse	Side adjacent to ∠A / Hypotenuse
ΔABC					
ΔADE					
ΔAFG					
ΔAHJ					

Nested Triangles Activity

The Sine and Cosine Ratios

The short form for the **sine ratio** of angle A is ______

The short form for the **cosine ratio** of angle A is ______



sin A = ______ / ______

cos A = ______ / ______

The Sine & Cosine Ratios

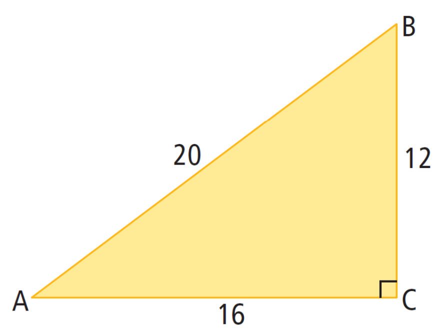
Example 1 Write Trigonometric Ratios

a) sin A

b) sin B

c) cos A

d) cos B



Example 1

Example 1 Write Trigonometric Ratios

a) sin A

$$\sin A = \frac{\textit{side opposite to } A}{\textit{hypotenuse}} = \frac{12}{20} = \frac{3}{5}$$

b) sin B

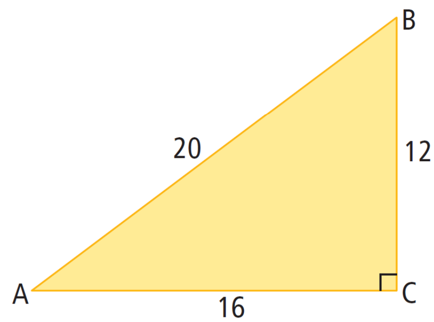
$$\sin B = \frac{\textit{side opposite to } B}{\textit{hypotenuse}} = \frac{16}{20} = \frac{4}{5}$$

c) cos A

$$\cos A = \frac{\textit{side adjacent to } A}{\textit{hypotenuse}} = \frac{16}{20} = \frac{4}{5}$$

d) cos B

$$\cos B = \frac{\textit{side adjacent to } B}{\textit{hypotenuse}} = \frac{12}{20} = \frac{3}{5}$$



Example 1 Solution

Example 2 Evaluate Trigonometric Ratios

The primary trigonometric ratios and their inverses can be evaluated using technology.

a) Evaluate each ratio, to four decimal places.

sin 42° cos 68°

b) Determine each angle measure, to the nearest degree.

sin A = 0.4771 cos B = 0.7225

Example 2

Example 2 Evaluate Trigonometric Ratios

The primary trigonometric ratios and their inverses can be evaluated using technology.

a) Evaluate each ratio, to four decimal places.

sin 42° cos 68°

$\sin 42° \simeq 0.6691$ $\cos 68° \simeq 0.3746$

b) Determine each angle measure, to the nearest degree.

sin A = 0.4771 cos B = 0.7225

$A = \sin^{-1}(0.4471)$ $B = \cos^{-1}(0.7225)$

$A \simeq 27°$ $B \simeq 44°$

Example 2 Solution

Example 3 Solving for an Angle

In the World Cup Downhill held at Panorama Mountain Village in British Columbia, the skiers raced 3514 m down the mountain. If the vertical height of the course was 984 m, determine the average angle of the ski course with the ground. Express your answer to the nearest tenth of a degree.

Step One:	Label
Step Two:	Formula
Step Three:	Substitute
Step Three:	Cross-multiply

Example 3

Example 3 Solving for an Angle

In the World Cup Downhill held at Panorama Mountain Village in British Columbia, the skiers raced 3514 m down the mountain. If the vertical height of the course was 984 m, determine the average angle of the ski course with the ground. Express your answer to the nearest tenth of a degree.

$$\sin\theta = \frac{984}{3514}$$

$$\theta = \sin^{-1}\left(\frac{984}{3514}\right)$$

$$\theta \simeq 16.3°$$

Step One: Label
Step Two: Formula
Step Three: Substitute
Step Three: Cross-multiply

Example 3 Solution

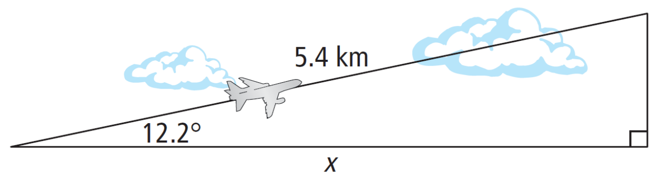
Your Turn

A guy wire supporting a cell tower is 24 m long. If the wire is attached at a height of 17 m up the tower, determine the angle that the guy wire forms with the ground.

Your Turn 3

Example 4 Solving for a Side

A pilot starts his takeoff and climbs steadily at an angle of 12.2°. Determine the horizontal distance the plane has travelled when it has climbed 5.4 km along its flight path. Express your answer to the nearest tenth of a kilometre.



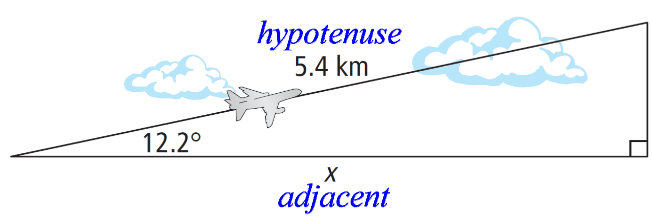
Example 4

Example 4 Solving for a Side

A pilot starts his takeoff and climbs steadily at an angle of 12.2°. Determine the horizontal distance the plane has travelled when it has climbed 5.4 km along its flight path. Express your answer to the nearest tenth of a kilometre.

$$\frac{\cos 12.2°}{1} = \frac{x}{5.4}$$

$$x \simeq 5.3$$



The plane has traveled a horizontal distance of about 5.3 km.

Example 4 Solution

Your Turn

Determine the height of a kite above the ground if the kite string extends 480 m from the ground and makes an angle of 62° with the ground. Express your answer to the nearest tenth of a metre.

Your Turn 4

End of Lesson

Assignment: Page 120 # 1-7, 10, 14
Challenge: Page 120 # 15

Assignment 2

Lesson 3 - Solving Right Triangles

Measurement

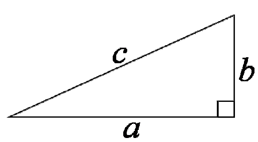
4. Develop and apply the primary trigonometric ratios (sine, cosine, tangent) to solve problems that involve right triangles.

Lesson 3: Solving Right Triangles

Definition

To *solve a triangle* means to [redacted] the measure of all its unknown [redacted] and unknown [redacted]
We have four ways to find unknown values of a triangle, the trigonometric ratios, [redacted], [redacted] and [redacted] as well as the [redacted].

Recall: The formula for the *Pythagorean Theorem* is
$a^2 + b^2 = c^2$ where ***a*** and ***b*** are the side legs of a right triangle and ***c*** is the hypotenuse.



Recall: Some people use an acronym to help them remember the trigonometric ratios. If it helps you, you may use it as well.

$\sin\theta = \frac{opposite}{hypotenuse}$ $\cos\theta = \frac{adjacent}{hypotenuse}$ $\tan\theta = \frac{opposite}{adjacent}$

SOH CAH TOA

Solving a Triangle Definition

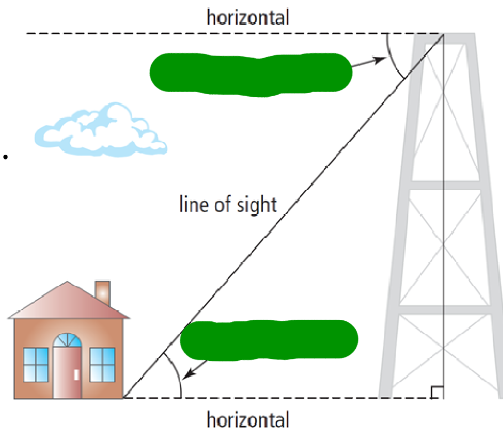
Definitions

The *Angle of Elevation* is the angle formed between the horizontal and a line of sight [redacted] the horizontal.

The ***Angle of Depression*** is the angle formed between the horizontal and a line of sight [redacted] the horizontal.

Label these angles on the diagram.

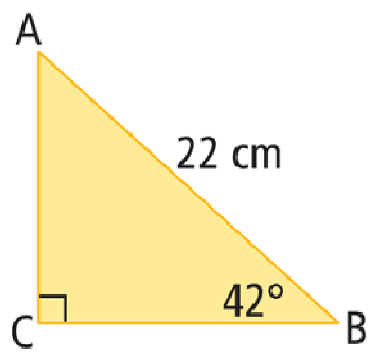
Note: The angles of elevation and depression are always [redacted].



Angle of Elevation & Depression Definitions

Example 1 Solve a Triangle

Solve Δ ABC. Give the side measures to the nearest tenth and angle measures to the nearest degree.



Example 1

Example 1 Solve a Triangle

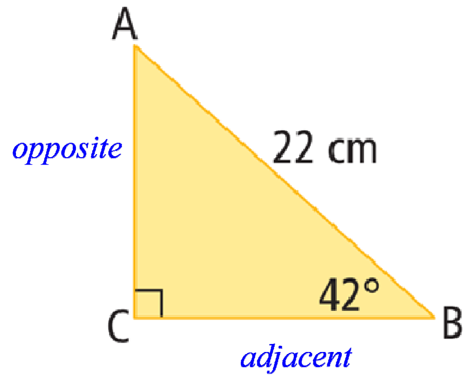
Solve Δ ABC. Give the side measures to the nearest tenth and angle measures to the nearest degree.

$$\frac{Sin\ 42°}{1} = \frac{AC}{22}$$

$$AC \simeq 15\ cm$$

$$\frac{Cos\ 42°}{1} = \frac{BC}{22}$$

$$BC \simeq 16\ cm$$



$$\angle A = 90° - \angle B$$

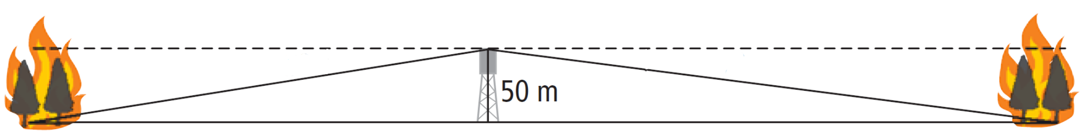
$$\angle A = 90° - 42°$$

$$\angle A = 48°$$

Example 1 Solution

Example 2 Solving a Problem Using Trigonometry

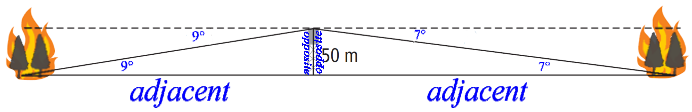
From a height of 50 m in his fire tower near Francois Lake, BC, a ranger observes the beginnings of two fires. One fire is due west at an angle of depression of 9°. The other fire is due east at an angle of depression of 7°. Determine the distance between the two fires, to the nearest metre.



Example 2

Example 2 Solving a Problem Using Trigonometry

From a height of 50 m in his fire tower near Francois Lake, BC, a ranger observes the beginnings of two fires. One fire is due west at an angle of depression of 9°. The other fire is due east at an angle of depression of 7°. Determine the distance between the two fires, to the nearest metre.



$$\frac{\tan 9°}{1} = \frac{50}{y}$$

$$y \simeq 315.69m$$

$$\frac{\tan 7°}{1} = \frac{50}{x}$$

$$x \simeq 407.22m$$

$$\text{Distance} = x + y$$

$$d = 315.69 + 407.22$$

$$d \simeq 723\,m$$

The fires are about 723 m apart.

Example 2 Solution

End of Lesson

Assignment: Page 131 #1ac, 2a, 4ab, 5ab, 7, 12

Challenge: Page 131 #15

Assignment 3