

**In this unit: Lesson 5.1 Multiplying Polynomials**

**Lesson 5.2 Common Factors**

**Lesson 5.3 Factoring Trinomials**

**Lesson 5.4 Factoring Special Trinomials**

|  |  |
| --- | --- |
| **General Goal** | **Specific Goal** |
| Develop algebraic reasoning and  number sense. | 4. Demonstrate an understanding of the multiplication of polynomial expressions (limited to monomials, binomials and trinomials), concretely, pictorially and symbolically.  5. Demonstrate an understanding of common factors and trinomial factoring, concretely, pictorially and symbolically. |

**5.1 Multiplying Polynomials**

Together with a partner define the following without looking in your text:

* Prime Numbers-
* Composite Numbers-
* Prime Factors-
* Greatest Common Factors-
* Lowest Common Multiples-
* Perfect Squares-
* Monomial-
* Binomial-
* Trinomial-

The Locker Problem

In a middle school, there is a row of 100 closed lockers numbered 1 to 100. A student

goes through the row and opens every locker. A second student goes through the row and for

every second locker if it is closed, she opens it and if it is open, she closes it. A third student

does the same thing for every third, a fourth for every fourth locker and so on, all the way to the

100th locker. The goal of the problem is to determine which lockers will be open at the end of

the process.

Questions:

In words, explain your thinking to the following problems clearly. Be sure to use

appropriate mathematical language and models:

• Which lockers remain open after the 100th student has passed?

• If there were 500 students and lockers, which lockers remain open after the 500th student

has passed?

• If there were 1000 students and lockers, which lockers remain open after the 1000th

student has passed?

• What is the rule for any number of students and lockers? Explain why your rule works.

• Which lockers were touched by only two students? How do you know?

Warm-up Activity

**1.** Complete the multiplication (13)(11).

**2. a)** You can express 13 as the sum 10 + 3, and 11 as the sum 10 + 1.

Complete an area model with the dimensions 10 + 3 and 10 + 1.

**b)** How does your model show the factors 10 + 3 and 10 + 1?

**c)** What is the product?

**Algebra Tiles**

1. Draw the Algebra tile used to represent each of the following:

**1 x x2**

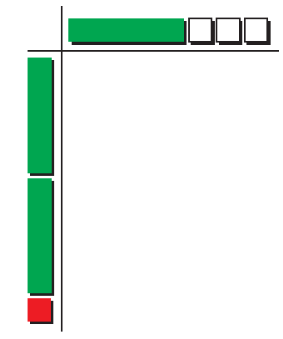


1. Use algebra tiles to determine the product (*x* + 3)(*x* + 1).
2. Sketch your model.
3. How does your model show the factors *x* + 3 and *x* + 1?
4. What is the product?
5. Use algebra tiles to determine each product.
   1. 
   2. 

Example 1: Multiplying Binomials

Multiply 

Method 1: Algebra Tiles Method 2: Distributive Property





Example 2 Multiply a Binomial by a Trinomial



Example 3 Combining Operations



End of Lesson

**Assignment:** Page 209 #1-4 odd letters, #5, #6 b,d, #10, 11, 12, 16

**Challenge:** Page 209 #17

**Lesson 5.2 Common Factors**

Recall: A *term* is a product of numbers and variables. Terms are separated by addition or subtraction signs.

Example: State the number of terms in the following polynomials.

1. 
2. 
3. 
4. 
5. 
6. 
7. 

Recall: Simplify or Expand means to multiply and remove the brackets.

For example:

Expand 

Simplify 

Reversing this process is called dividing or factoring.

For example:

Factor  Factor 

The process of factoring with polynomials is similar to factoring with numbers.

**Example**

Determine the GCF of 16*x*2*y* and 24*x*2*y*3.

**Solution**

**Method 1: Use Prime Factorization**

List the prime factorization of the numerical coefficients.

16 =

24 =

Numerical GCF

List the prime factorization of the variables.

*x*2*y* =

*x*2*y*3 =

Variable GCF =

Therefore, the GCF of 16*x*2*y* and 24*x*2*y*3 is

**Method 2: List the Factors**

Write the factors of each term

16*x*2*y*:

24*x*2*y*3:

The greatest common factors are

Therefore, the GCF of 16*x*2*y* and 24*x*2*y*3 is

**Your Turn**

Determine the GCF of each pair of terms.

1. 5*m*2*n* and 15*mn*2 **b)** 48*ab*3*c* and 36*a*2*b*2*c*2

**Example 2**

Write 7 *a2b* - 28*ab* + 14*a b2* in factored form.

**Solution**

Ask the questions: How many terms are there? What is common in each term?

Identify the GCF of the numerical coefficients by listing the prime factorization for each coefficient.

7 = 7

28 = (2)(2)(7)

14 = (2)(7)

The GCF is 7.

Identify the GCF of the variables.

*a2b* =

*ab* =

*ab2* =

The GCF is

Therefore, the GCF of 7 *a2b* - 28*ab* + 14*ab2*  is

Divide each term by the GCF.

7 *a2b* - 28*ab* + 14*a b2* =

Check:

Multiply.

**Your Turn**

Write each polynomial in factored form.

**a)** 27*r* 2*s*2 - 18*r*3*s*2 -36*rs*3

**b)** 4*np*2 + 10*n*4*p* - 12*n*3*p*

Recall terms are separated by plus or minus signs.

State the number of terms in each polynomial:

* 
* 
* 
* 
* 
* 

**Example 3**

Write the expression in factored form.

3*x*(*x* - 4) + 5(*x* - 4)

**Solution**

How many terms are there? What is common in each term?

**Your Turn**

Write each expression in factored form.

1. 4(*x* + 5) - 3*x*(*x* + 5)
2. 12b(a – 7) + (a - 7)
3. (x – 6) (5x+2)+(x+3) (5x+2)
4. 

**Factoring by Grouping**

**Example 4**

When there are 4 or more terms Factoring by Grouping can be used.

Write the expression in factored form.

*y*2 + 8*xy* + 2*y* + 16*x*

**Solution**

How many terms are there? What is common in each term?

Check:

**Your Turn**

1. *a*2 + 8*ab* + 2*a* + 16*b*
2. *ax + a + bx + b*
3. *x2 + y – xy – x*
4. x3 + x +2x2 + 2
5. yz – xy + y2 – xz

**Common Factoring With Fractions**

When the polynomial being factored contains a fraction, it is advantageous to factor out the fraction so the remaining factor has integral coefficients.

**Example 5**

Factor the following:

1. 

**Your Turn**

1. 
2. 

End of Lesson

**Assignment:** Page 220#2 odd letters, #4-7 odd letters, 8, 9, 11-12 odd letters, 16

**Challenge**: Page 220 #19

**Lesson 5.3 Factoring Trinomials**

A rectangle can have an area that is a trinomial. By finding the dimensions of the rectangle, you are reversing the process of multiplying two binomials. This process is called *factoring*.

You can factor a trinomial of the form *x*2 + *bx* + *c* and the form *ax*2 + *bx* + *c* by studying patterns.

Observe patterns that result from multiplying two binomials.

Multiply *x* + 2 and *x* + 3.

(*x* + 2)(*x* + 3)

= *x*2 + 2*x+* 3*x* + (2)(3)

= *x*2 + 5*x* + 6

Note that \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Example 1 Factor Trinomials of the Form *ax*2** + ***bx*** + ***c*, *a*** = **1**

Factor, if possible.*x***2** + 5*x* + 4

**Solution**

**Method 1: Use Algebra Tiles**

Arrange one *x*2-tile, five *x*-tiles,

and four 1-tiles into a rectangle.

Then, add tiles to show the dimensions.

The dimensions of the rectangle are. Therefore, the factors are

**Method 2: Symbolic** Factor *x*2 + 5*x* + 4.

**Solution** The first term in each factor must be x.

(x ) ( x )

To find the last terms in each factor, think of two numbers that multiply to equal the coefficient of the last term and add to equal the coefficient of the middle term

\_\_\_\_ x \_\_\_\_\_ = 4 \_\_\_\_ + \_\_\_\_\_ = 5

The two numbers are 4 and 1.

The factors will be (x + 1) (x + 4)

Check by multiplying:

**Example 2**

Factor if possible.

1. *x***2** - 29*x* + 28

( ) ( )

\_\_\_\_ x \_\_\_\_\_ = \_\_\_\_ + \_\_\_\_\_ =

Therefore, the factors are

1. *x***2** + 3*xy* - 18*y*2

(x y) (x y)

\_\_\_\_ x \_\_\_\_\_ = \_\_\_\_ + \_\_\_\_\_ =

Therefore, the factors are

1. *x***2** + 4*x* + 6

( ) ( )

\_\_\_\_ x \_\_\_\_\_ = \_\_\_\_ + \_\_\_\_\_ =

**Your Turn**

Factor, if possible.

1. *x*2 + 7*x* + 10 **b)** *r*2 - 10*rs* + 9*s*2

**Example 3 Factor Trinomials of the Form *ax*2 + *bx* + *c*, *a* ≠ 1**

Factor, if possible.

3*x*2 + 8*x* + 4

**Solution**

First, check for a GCF. The GCF of the polynomial 3*x*2 + 8*x* + 4 is 1.

**Method 1: Use Algebra Tiles**

Arrange three *x*2-tiles, eight *x*-tiles, and four 1-tiles into a rectangle. Then, add tiles to show the dimensions.

The dimensions of the resulting rectangle are

Check:

Multiply.

(3*x* + 2)(*x* + 2) =

**Method 2: The Cross or Diamond Method**

**Recall: (3*x* + 2)(*x* + 5)**

**= 3*x*2 + 15*x* + 2*x* + 10**

**= 3*x*2 + 17*x* + 10**

**Note that the sum of 15*x* + 2*x* is the middle term, 17*x.***

**The product of these two numbers is 30*x*2.**

**This is the same as the product of the first and last terms of the trinomial. ( 3*x*2 x 10 = 30*x*2)**

Therefore, to factor 3*x*2 + 17*x* + 10, look for two numbers that have

a product of 30 and a sum of 17.

A cross can be used to help organize these numbers when factoring.

First times Last

Middle

Side

Side

\_\_\_\_ x \_\_\_\_ = Top

\_\_\_\_ + \_\_\_\_\_ = Bottom

*These numbers become the side numbers on the cross.*

( ) ( )

The first terms of each bracket will be factors of the first term of the trinomial.

( ) ( )

Product equals one side number

Product equals other side number

Once all numbers are filled in on the cross, step away from the cross and use the brackets to finish factoring.

**Example 3 continued**

**Factor 3*x*2 + 17*x* + 10**

( ) (  )

*The first terms of each bracket will be factors of the first term of the trinomial.*

( ) ( )

Product equals one side number

Product equals other side number

\_15\_ x \_2\_ = 30

\_15\_ + \_2\_ = 17

*These numbers become the side numbers on the cross.*

The factors of **3*x*2 + 17*x* + 10 are**

**Example 4**

Factor, if possible 3*x*2 - 8*x* + 4

Solution

( ) ( )

**Example 5**

Factor, if possible **12*y*2 - 5*y* - 3**

Solution

There are 3 possible pairs of factors for the first terms in the brackets. 1 x 12, 2 x 6, and 3 x 4. Use the side numbers to help you decide which pair to use. Since 3 and 4 divide evenly into -9 and 4, the chosen pair will be 3 and 4.

( ) ( )

**Example 6**

Factor, if possible ***6x2 – 5xy +y2***

( y ) ( y )

**Your Turn**

Factor, if possible.

1. *2a2 - 7a - 15*
2. *4x2 - 12x + 5*
3. *15a2 - 14ab - 8b2*
4. *10a2 + 11a + 3*
5. *6a3 + 26a2 + 20a*

End of Lesson

**Assignment:** Page 234 #1-12 odd letters, 13

**Challenge**: Page 23 #18

Lesson 5.4 Factoring Special Trinomials

You will find patterns helpful in factoring polynomials with special products. These include differences of squares and perfect square trinomials.

**Difference of Squares**

When you multiply the sum and the difference of two terms, the product will be a difference of squares.

In a difference of squares:

• the expression is a binomial

• the first term is a perfect square: *u* 2

• the last term is a perfect square: *v* 2

• the operation between the two terms is subtraction

A difference of squares, *\_\_\_\_\_\_\_\_\_\_\_\_\_\_*, can be factored into \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Perfect Square Trinomial**

When you square a binomial, the result is a perfect square trinomial.

In a perfect square trinomial

• the first term is a perfect square: *x*2

• the last term is a perfect square: 52

• the middle term is twice the product of the square root of the first term and the square root of the last term:

(2)(*x*)(5) = 10*x*

**Example 1 Factor a Difference of Squares**

Factor each binomial, if possible.

**a)** *x*2 – 9

The binomial *x*2 - 9 is a difference of squares.

The first term is a perfect square: *x*2

The last term is a perfect square: 32

The operation is subtraction.

**b) -**16*c*2+ 25*a*2

The binomial 25*a*2- 16*c*2 is a difference of squares.

The first term is a perfect square: (5*a*) 2

The last term is a perfect square: (4*c*) 2

The operation is subtraction.

**c)** *m*2 + 16

**d)** 7*g*3*h*2 - 28*g*5

First, factor out the GCF from 7*g*3*h*2 - 28*g*5

.

The binomial is a difference of squares.

The first term is a perfect square: *h*2

The last term is a perfect square: (2*g*)2

The operation is subtraction.

**Your Turn**

Factor each binomial, if possible.

1. 49*a*2 – 25
2. 125*x*2 - 40*y*2
3. 9*p*2*q*2 – 25

**Example 2 Factor Perfect Square Trinomials**

Factor each trinomial, if possible.

1. *x*2 + 6*x* + 9

The trinomial *x*2 + 6*x* + 9 is a perfect square.

The first term is a perfect square: *x*2

The last term is a perfect square: 32

The middle term is twice the product of the square root of the first term and the square root of the last term: (2)(*x*)(3) = 6*x*

The trinomial is of the form (*ax*) 2+ 2*abx* + *b*2.

1. 2*x*2 - 44*x* + 242

The first term in the brackets is a perfect square: *x*2

The last term in the brackets is a perfect square: 112

The middle term is twice the product of the square root of the first term and the square root of the last term: (2)(*x*)(11) = 22*x*

1. *c*2 - 12*x* – 36

**Your Turn**

Factor each trinomial, if possible.

1. *x*2 - 24*x* + 144
2. *y*2 - 18*y* – 81
3. 3*b*2 + 24*b* + 48

End of Lesson

**Assignment:** Page 246 # 5-9 odd letters, 14, 17, 19, 20