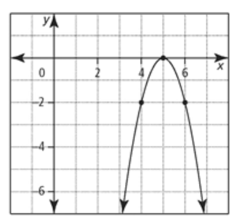
Unit 3- Quadractics Functions and Equations

Lesson 3.2 Quadratic Functions in Standard Form

Specific Outcome
2. Analyze quadratic functions of the form $y=ax^2+bx+c$ to identify characteristics of the corresponding graph, including: • vertex • domain and range • direction of opening • axis of symmetry • x- and y-intercepts and to solve problems.

Two Questions - Review

1. Determine a quadratic function in vertex form for the following parabola.



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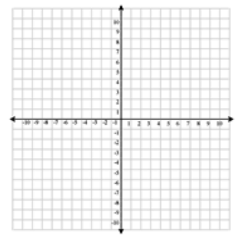
2. Sketch the graph of the following functions. **Identify the x- and y- intercepts if there are any.**

$y = -(x-2)^2 + 1$



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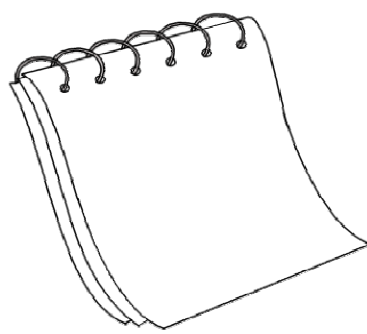
Quadratic Functions in Standard Form

The standard form of a quadratic function is **$f(x) = ax^2 + bx + c$ or**
$y = ax^2 + bx + c$ where *a, b* and *c* are real numbers and **$a \neq 0$:**

- **a** determines the width of the graph (large a means narrow, small a means wide)
- **a** determine which direction the parabola opens (positive a opens up, negative a opens down)
- **b** influences the position of the graph
- **c** is the y-intercept of the graph



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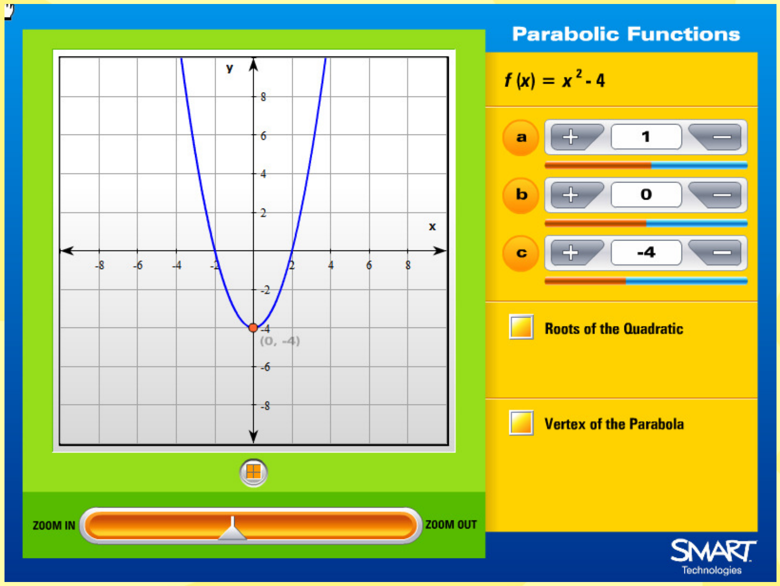

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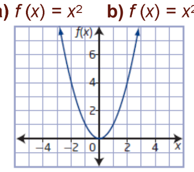
Example 1

Identify Characteristics of a Quadratic Function in Standard Form

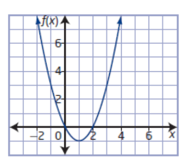
For each graph of a quadratic function, identify the following:

- the direction of opening
- the coordinates of the vertex
- the maximum or minimum value
- the equation of the axis of symmetry
- the x-intercepts and y-intercept
- the domain and range

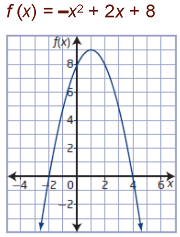
a) $f(x) = x^2$



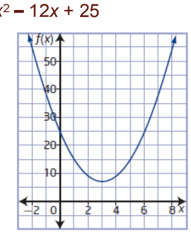
b) $f(x) = x^2 - 2x$



c) $f(x) = -x^2 + 2x + 8$



d) $f(x) = 2x^2 - 12x + 25$



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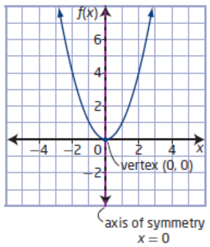
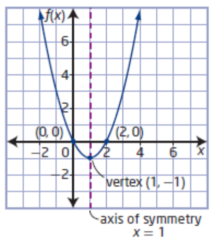
3.2 Example 1 Continued

1 **a)** $f(x) = x^2$



- opens upward
- vertex: (0, 0)
- minimum value of y of 0 when $x = 0$
- axis of symmetry: $x = 0$
- y-intercept occurs at (0, 0) and has a value of 0
- x-intercept occurs at (0, 0) and has a value of 0
- domain: all real numbers, or $\{x \mid x \in R\}$
- range: all real numbers greater than or equal to 0, or $\{y \mid y \geq 0, y \in R\}$

2 **b)** $f(x) = x^2 - 2x$



- opens upward
- vertex: (1, –1)
- minimum value of y of –1 when $x = 1$
- axis of symmetry: $x = 1$
- y-intercept occurs at (0, 0) and has a value of 0
- x-intercepts occur at (0, 0) and (2, 0) and have values of 0 and 2
- domain: all real numbers, or $\{x \mid x \in R\}$
- range: all real numbers greater than or equal to –1, or $\{y \mid y \geq -1, y \in R\}$

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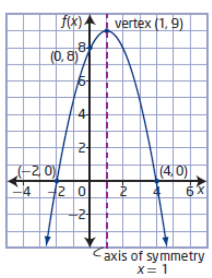
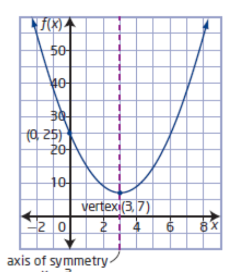
3.2 Example 1 Continued

3 **c)** $f(x) = -x^2 + 2x + 8$



- opens downward
- vertex: (1, 9)
- maximum value of y of 9 when $x = 1$
- axis of symmetry: $x = 1$
- y-intercept occurs at (0, 8) and has a value of 8
- x-intercepts occur at (–2, 0) and (4, 0) and have values of –2 and 4
- domain: all real numbers, or $\{x \mid x \in R\}$
- range: all real numbers less than or equal to 9, or $\{y \mid y \leq 9, y \in R\}$

4 **d)** $f(x) = 2x^2 - 12x + 25$



- opens upward
- vertex: (3, 7)
- minimum value of y of 7 when $x = 3$
- axis of symmetry: $x = 3$
- y-intercept occurs at (0, 25) and has a value of 25
- no x-intercepts
- domain: all real numbers, or $\{x \mid x \in R\}$
- range: all real numbers greater than or equal to 7, or $\{y \mid y \geq 7, y \in R\}$

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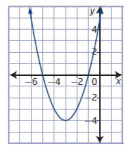
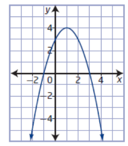
Example 1: Your Turn

For each quadratic function, identify the following:
- the direction of opening
- the coordinates of the vertex
- the maximum or minimum value
- the equation of the axis of symmetry
- the x-intercepts and y-intercept
- the domain and range

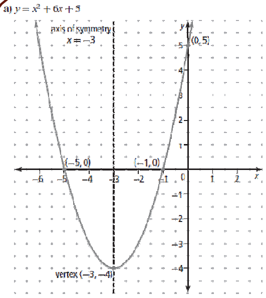
a) $y = x^2 + 6x + 5$



b) $y = -x^2 + 2x + 3$



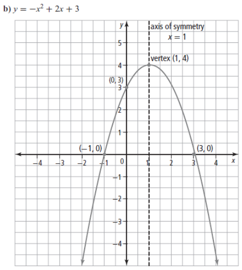
a) $y = x^2 + 6x + 5$



The graph opens upward. The coordinates of the vertex are $(-3, -4)$. The graph has a minimum value of -4 when x is -3. The equation of the axis of symmetry is $x = -3$. The x-intercepts occur at $(-5, 0)$ and $(-1, 0)$, and have values of -5 and -1 respectively. The y-intercept occurs at $(0, 5)$ and has a value of 5. The domain is the set of all real numbers or $\{x \mid x \in R\}$. The range is the set of real numbers where y is greater than or equal to -4, or $\{y \mid y \geq -4, y \in R\}$.

Answer Part b)

b) $y = -x^2 + 2x + 3$



The graph opens downward. The coordinates of the vertex are (1, The graph has a maximum value of 4 when x is 1. The equation o axis of symmetry is $x = 1$. The x-intercepts occur at $(-1, 0)$ and (and have values of -1 and 3 respectively. The y-intercept occurs $(0, 3)$ and has a value of 3. The domain is the set of all real numb or $\{x \mid x \in R\}$. The range is the set of real numbers where y is les than or equal to 4, or $\{y \mid y \leq 4, y \in R\}$.

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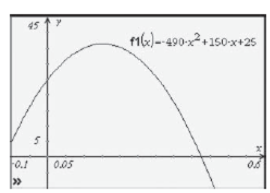
Example 2

Analysing a Quadratic Function

A frog sitting on a rock jumps into a pond. The height, h, in centimetres, of the frog above the surface of the water as a function of time, t, in seconds, since it jumped can be modelled by the function $h(t) = -490t^2 + 150t + 25$. Where appropriate, answer the following questions to the nearest tenth.

a) Graph the function.
b) What is the y-intercept? What does it represent in this situation?
c) What maximum height does the frog reach? When does it reach that height?
d) When does the frog hit the surface of the water?
e) What are the domain and range in this situation?
f) How high is the frog 0.25 s after it jumps?

1 **a) Method 1: Use a Graphing Calculator**
Enter the function and adjust the dimensions of the graph until the vertex and intercepts are visible.



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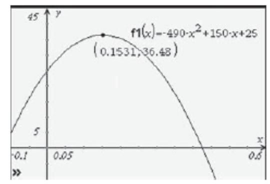
3.2 Example 2 Continued

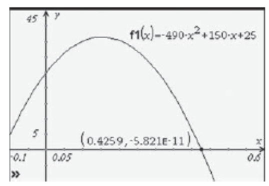
2 **b)** The graph shows that the y-intercept is 25. This is the value of h at $t = 0$. It represents the initial height, 25 cm, from which the frog jumped.

The y-intercept of the graph of $h(t) = -490t^2 + 150t + 25$ is equal to the value of the constant term, 25.

3 **c)** The coordinates of the vertex represent the time and height of the frog at its maximum point during the jump. The graph shows that after approximately 0.2 s, the frog achieves a maximum height of approximately 36.5 cm.



4 **d)** The positive x-intercept represents the time at which the height is 0 cm, or when the frog hits the water. The graph shows that the frog hits the water after approximately 0.4 s.



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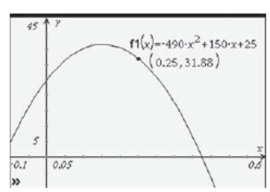
3.2 Example 2 Continued

5 **e)** The domain is the set of all possible values of the independent variable, or time.
The range is the set of all possible values of the dependent variable, or height.
The values of time and height cannot be negative in this situation.
The domain is the set of all real numbers from 0 to approximately 0.4, or $\{t \mid 0 \le t \le 0.4,\ t \in R\}$.
The range is the set of all real numbers from 0 to approximately 36.5, or $\{h \mid 0 \le h \le 36.5,\ h \in R\}$.

6 **f)** The height of the frog after 0.25 s is the h-coordinate when t is 0.25. The graph shows that after 0.25 s, the height of the frog is approximately 31.9 cm. You can also determine the height after 0.25 s by substituting 0.25 for t in $h(t) = -490t^2 + 150t + 25$.



$h(t) = -490t^2 + 150t + 25$
$h(0.25) = -490(0.25)^2 + 150(0.25) + 25$
$h(0.25) = -30.625 + 37.5 + 25$
$h(0.25) = 31.875$

The height of the frog after 0.25 s is approximately 31.9 cm.

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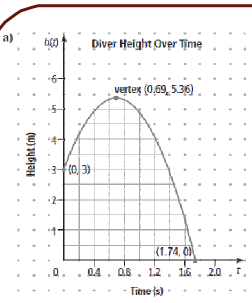
Example 2: Your Turn

A diver jumps from a 3-m springboard with an initial vertical velocity of 6.8 m/s. Her height, *h*, in metres, above the water *t* seconds after leaving the diving board can be modelled by the function $h(t) = -4.9t^2 + 6.8t + 3$.

a) Graph the function.
b) What does the *y*-intercept represent?
c) What maximum height does the diver reach? When does she reach that height?
d) How long does it take before the diver hits the water?
e) What domain and range are appropriate in this situation?
f) What is the height of the diver 0.6 s after leaving the board?

Answer

a)



b) height of the diving board

c) The diver achieves a maximum height of 5.36 m, 0.69 s after leaving the board.

d) The diver contacts the water 1.74 s after leaving the diving board.

e) domain $\{t \mid 0 \le t \le 1.74, t \in \mathrm{R}\}$; range $\{h \mid 0 \le h \le 5.36, h \in \mathrm{R}\}$

f) $h(0.6) = -4.9(0.6)^2 + 6.8(0.6) + 3 = 5.32$. The diver has a height of 5.32 m, 0.6 s after leaving the board.

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Example 3

Write a Quadratic Function to Model a Situation

A rancher has 100 m of fencing available to build a rectangular corral.

a) Write a quadratic function in standard form to represent the area of the corral.
b) What are the coordinates of the vertex? What does the vertex represent in this situation?
c) Sketch the graph for the function you determined in part a).
d) Determine the domain and range for this situation.
e) Identify any assumptions you made in modelling this situation mathematically.

1 **a)** Let *l* represent the length, *w* represent the width, and *A* represent the area of the corral.

The formula $A = lw$ has three variables. To create a function for the area in terms of the width alone, you can use an expression for the length in terms of the width to eliminate the length. The formula for the perimeter of the corral is $P = 2l + 2w$, which gives the equation $2l + 2w = 100$. Solving for *l* gives $l = 50 - w$.

$A = lw$
$A = (50 - w)(w)$
$A = 50w - w^2$

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3.2 Example 2 Continued

b) Use the equation $x = p$ to determine the x-coordinate of the vertex.

$$x = \frac{-b}{2a}$$

$$x = \frac{-50}{2(-1)}$$

$$x = 25$$

Substitute the x-coordinate of the vertex into the function to determine the y-coordinate.

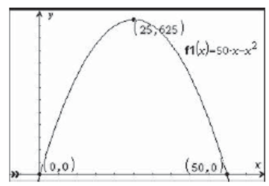
$y = 50x - x^2$

$y = 50(25) - (25)^2$

$y = 625$

The vertex is located at (25, 625). The y-coordinate of the vertex represents the maximum area of the rectangle. The x-coordinate represents the width when this occurs.

c) For the function $f(x) = 50x - x^2$, the y-intercept is the point (0, 0). Using the axis of symmetry, a point symmetric to the y-intercept is (50, 0). Sketch the parabola through these points and the vertex (25, 625).



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3.2 Example 2 Continued

d) Negative widths, lengths, and areas do not have any meaning in this situation, so the domain and range are restricted.
The width is any real number from 0 to 50.
The domain is $\{w \mid 0 \le w \le 50,\ w \in R\}$.
The area is any real number from 0 to 625.
The range is $\{A \mid 0 \le A \le 625,\ A \in R\}$.

e) The quadratic function written in part a) assumes that the rancher will use all of the fencing to make the corral. It also assumes that any width or length from 0 m to 50 m is possible. In reality, there may be other limitations on the dimensions of the corral, such as the available area and landscape of the location on the rancher's property.

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Example 3: Your Turn

At a children's music festival, the organizers are roping off a rectangular area for stroller parking. There is 160 m of rope available to create the perimeter.

a) Write a quadratic function in standard form to represent the area for the stroller parking.
b) What are the coordinates of the vertex? What does the vertex represent in this situation?
c) Sketch the graph for the function you determined in part a).
d) Determine the domain and range for this situation.
e) Identify any assumptions you made.

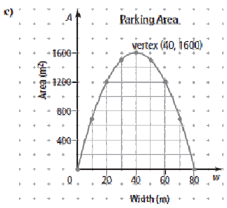
Answer

a) Since $P = 2l + 2w$ and $P = 160$ m, then $l = 80 - w$ and $A = -w^2 + 80w$.

$160 = 2l + 2w$	$A = lw$
$160 = 2(l + w)$	$A = (80 - w)w$
$80 = l + w$	$A = 80w - w^2$
$l = 80 - w$	

b) Use $x = \frac{-b}{2a}$ to find the x-coordinate of the vertex. Then, substitute that value into the function to find the y-coordinate of the vertex. The vertex is located at (40, 1600). The vertex represents the maximum area of 1600 m^2 when the dimensions of the stroller park are 40 m by 40 m.

c)



d) domain $\{w \mid 0 \le w \le 80, w \in R\}$; range $\{A \mid 0 \le A \le 1600, A \in R\}$

e) Assumptions include:
- All 160 m of rope is used to rope off the stroller park.
- Any length or width from 0 m to 80 m is possible.

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