Lesson 2 - Integral Exponents

Algebra and Number

2. It is expected that students will:

• Demonstrate an understanding of powers with integral and rational exponents.

Investigate Negative Exponents

1. On a sheet of paper, draw a line 16 cm long and mark it as shown.

2. Mark a point halfway between 0 and 16. Label the point with its value and its equivalent value in exponential form (2^x). Repeat this procedure until you reach a value of 1 cm.

a) How many times did you halve the line segment to reach 1 cm?

b) What do you notice about the exponents as you keep reducing the line segment by half?

3. a) Mark the halfway point between 0 and 1. What fraction does this represent?

b) Using the pattern established in step 2, what is the exponential form of the fraction?

c) Halve the remaining line segment two more times.

4. Use a table to summarize the line segment lengths and the equivalent exponential form in base 2.

5. Reflect and Respond
a) Describe the pattern you observe in the exponents as the distance is halved.

b) Is there a way to rewrite each fraction so that it is expressed as a power with a positive exponent? Try it.
Compare this form to the equivalent power with a negative exponent. What is the pattern?

Exponent Law	Example
Note that a and b are rational or variable bases and m and n are integral exponents.	
Product of Powers $(a^m)(a^n) = a^{m+n}$	$(3^{-2})(3^4) = 3^{-2+4}$ $= 3^2$
Quotient of Powers $\frac{a^m}{a^n} = a^{m-n}, a \neq 0$	$\frac{x^3}{x^{-5}} = x^{3-(-5)}$ $= x^8$
Power of a Power $(a^m)^n = a^{mn}$	$(0.75^4)^{-2} = 0.75^{(4)(-2)}$ $= 0.75^{-8}$ or $\frac{1}{0.75^8}$
Power of a Product $(ab)^m = (a^m)(b^m)$	$(4z)^{-3} = \frac{1}{(4z)^3}$ or $\frac{1}{64z^3}$
Power of a Quotient $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}, b \neq 0$	$\left(\frac{t}{3}\right)^{-2} = \left(\frac{3}{t}\right)^2$ $= \frac{3^2}{t^2}$ $= \frac{9}{t^2}$
Zero Exponent $a^0 = 1, a \neq 0$	$(4y^2)^0 = 1$ $-(4y^2)^0 = -1$

To simplify expressions with integral exponents, you can use the following principle as well as the exponent laws.

A power with a negative exponent can be written as a power with a positive exponent.

- $a^{-n} = \frac{1}{a^n}, a \neq 0$ $\quad 2^{-3} = \frac{1}{2^3}$
- $\frac{1}{a^{-n}} = a^n, a \neq 0$ $\quad \frac{1}{2^{-3}} = 2^3$

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Chapter 4

Integral Exponents

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Write each expression with positive exponents.

a) 2^{-1}

b) 3^{-2}

c) $(4^{6})(4^{-3})$

d) $(5^{-8})(5^{2})$

e) $\dfrac{6^{-5}}{6^{2}}$

f) $\dfrac{7^{-2}}{7^{5}}$

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Example 1 Multiply or Divide Powers With the Same Base

Write each product or quotient as a power with a single exponent.

a) $(0.8^{-2})(0.8^{-4})$

Method 1: Add the Exponents

0.8^{-2+-4}

0.8^{-6}

$\frac{1}{0.8^{6}}$

Method 2: Use Positive Exponents

$(0.8)^{-2}(0.8)^{-4}$

$\frac{1}{(0.8)^{2}} \frac{1}{(0.8)^{4}}$

$\frac{1}{(0.8)^{2+4}}$

$\frac{1}{(0.8)^{6}}$

b) $\dfrac{(2x)^3}{(2x)^{-2}}$

Method 1: Subtract the Exponents

$(2x)^{3-(-2)}$

$(2x)^{3+2}$

$(2x)^5$

Method 2: Use Positive Exponents

$(2x)^3\,(2x)^2$

$(2x)^{3+2}$

$(2x)^5$

Example 1 **Multiply or Divide Powers With the Same Base**

Write each product or quotient as a power with a single exponent.

a) $(0.8^{-2})(0.8^{-4})$

Method 1: Add the Exponents

$$(0.8^{-2})(0.8^{-4}) = 0.8^{-2+(-4)}$$
$$= 0.8^{-6}$$
$$= \left(\frac{8}{10}\right)^{-6} = \left(\frac{10}{8}\right)^{6}$$
$$= \left(\frac{5}{4}\right)^{6}$$

Method 2: Use Positive Exponents

$$(0.8^{-2})(0.8^{-4}) = \left(\frac{1}{0.8^2}\right)\left(\frac{1}{0.8^4}\right)$$
$$= \frac{1}{(0.8^2)(0.8^4)}$$
$$= \frac{1}{0.8^{(2+4)}}$$
$$= \frac{1}{0.8^6}$$

b) $\dfrac{(2x)^3}{(2x)^{-2}}$

Method 1: Subtract the Exponents

$$\frac{(2x)^3}{(2x)^{-2}} = (2x)^{3-(-2)}$$
$$= (2x)^5$$

Method 2: Use Positive Exponents

$$\frac{(2x)^3}{(2x)^{-2}} = (2x)^3(2x)^2$$
$$= (2x)^{3+2}$$
$$= (2x)^5$$

a) $(2^{-3})(2^{5}) \rightarrow \frac{1}{2^3}(2^5) = \frac{2^5}{2^3} = 2^{5-3} = 2^2$

$\downarrow$

$2^{-3+5} = 2^2$

b) $\frac{7^{-5}}{7^3} \rightarrow \frac{1}{7^5\ 7^3} = \frac{1}{7^8}$

$\downarrow$

$7^{-5-3} = 7^{-8} = \frac{1}{7^8}$

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c) $\dfrac{(-3.5)^4}{(-3.5)^{-3}} \rightarrow (-3.5)^4(-3.5)^3 = (-3.5)^{4+3}$

$= (-3.5)^7$

$\downarrow$

$(-3.5)^{4--3} = (-3.5)^{4+3} = (-3.5)^7$

d) $\dfrac{(3y)^2}{(3y)^{-6}} \rightarrow (3y)^2(3y)^6 = (3y)^{2+6} = (3y)^8$

$\swarrow$

$(3y)^{2--6} = (3y)^{2+6} = (3y)^8$

Example 2 Powers of Powers

Write each expression as a power with a single, positive exponent. Then, evaluate where possible.

a) $(4^3)^{-2}$

4^{-6}

$\frac{1}{4^6}$

b) $\left(\frac{2^4}{2^6}\right)^{-3}$

method 1

$(2^{4-6})^{-3}$

$(2^{-2})^{-3}$

2^6

method 2

$\frac{2^{-12}}{2^{-18}}$

$2^{-12-(-18)}$

$2^{-12+18} = 2^6$

c) $\left[\left(\frac{3}{4}\right)^{-2} \cdot \left(\frac{3}{4}\right)^{-2}\right]^{-2}$

$\left[\left(\frac{3}{4}\right)^{-2+(-2)}\right]^{-2}$

$\left(\left(\frac{3}{4}\right)^{-4}\right)^{-2}$

$\left(\frac{3}{4}\right)^{8}$

a) $(4^3)^{-2}$

Solution

a) Multiply the exponents. Then, rewrite as a positive exponent.

$$\begin{aligned}(4^3)^{-2} &= 4^{(3)(-2)} \\ &= 4^{-6} \\ &= \frac{1}{4^6} \\ &= \frac{1}{4096}\end{aligned}$$

b) $\left(\frac{2^4}{2^6}\right)^{-3}$

Solution

Method 1: Simplify Within the Brackets

Since the bases are the same, you can subtract the exponents. Raise the result to the exponent -3. Then, multiply.

$$\begin{aligned}\left(\frac{2^4}{2^6}\right)^{-3} &= [2^{(4-6)}]^{-3}\\ &= (2^{-2})^{-3}\\ &= 2^{(-2)(-3)}\\ &= 2^6\\ &= 64\end{aligned}$$

Method 2: Raise Each Power to an Exponent

Raise each power to the exponent -3. Then, divide the resulting powers by subtracting the exponents, since they have the same base.

$$\begin{aligned}\left(\frac{2^4}{2^6}\right)^{-3} &= \frac{(2^4)^{-3}}{(2^6)^{-3}}\\ &= \frac{2^{(4)(-3)}}{2^{(6)(-3)}}\\ &= \frac{2^{-12}}{2^{-18}}\\ &= 2^{-12-(-18)}\\ &= 2^6\\ &= 64\end{aligned}$$

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c) $\left[\left(\frac{3}{4}\right)^{-2}\left(\frac{3}{4}\right)^{-2}\right]^{-2}$

Solution

c) Add the exponents. Raise the resulting power to the exponent -2.

$$= \left[\left(\frac{3}{4}\right)^{2}\right]^{-2}$$

$$= \left(\frac{3}{4}\right)^{(2)(-2)}$$

$$= \left(\frac{3}{4}\right)^{-4}$$

$$= \frac{1}{\left(\frac{3}{4}\right)^{4}}$$

$$= \left(\frac{4}{3}\right)^{4}$$

$$= \frac{256}{81}$$

a) $\left[(0.6)^3(0.6)^{-3}\right]^{-5}$

$\left[(0.6)^{3+-3}\right]^{-5}$

$\left[(0.6)^0\right]^{-5}$

$0.6^0 = 1$

b) $\left[(t^{-4})(t^3)\right]^{-3}$

$\left(t^{-4+3}\right)^{-3}$

$\left(t^{-1}\right)^{-3}$

t^3

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c) $\left(\frac{x^6}{x^4}\right)^{-2}$

$\left(x^{6-4}\right)^{-2}$

$\left(x^2\right)^{-2}$

x^{-4}

$\frac{1}{x^4}$

d) $\left[\frac{(y^2)^0}{y^3}\right]^{-3}$

$\left(\frac{y^0}{y^3}\right)^{-3}$

$\left(y^{0-3}\right)^{-3}$

$\left(y^{-3}\right)^{-3} = y^9$

Example 3 Apply Powers With Integral Exponents

It is estimated that there are 117 billion grasshoppers in an area of 39 000 km^2 of Saskatchewan. Approximately how many grasshoppers are there per square kilometre?

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Solution

Method 1: Use Arithmetic

Divide the number of grasshoppers by the total area.

$$\text{grasshoppers per square kilometre} = \frac{117\,000\,000\,000}{39\,000}$$
$$= 3\,000\,000$$

There are approximately 3 000 000 grasshoppers per square kilometre.

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Method 2: Use Exponent Rules

Since you cannot enter numbers as large as 117 billion directly into most calculators, rewrite them using exponential form. Then, use the exponent rules to calculate the power of 10.

$$\text{grasshoppers per square kilometre} = \frac{(117)(10^9)}{(39)(10^3)}$$
$$= (3)(10^{(9-3)})$$
$$= (3)(10^6)$$

There are approximately 3 000 000 grasshoppers per square kilometre.

$$\frac{(117)(10^9)}{39(10^3)}$$
$$3(10^{9-3})$$
$$3(10^6)$$