**4.2 Factoring Quadratic Expressions**

**Specific Outcome:** Factor polynomial expressions of the form:

• *ax2*+*bx* + *c*, *a* ≠ 0

• *a2* *x2* –*b2* 2*y* , *a* ≠ 0, *b* ≠ 0

•*a*(*f* (*x*))2 + *b*(*f*(*x*))2 +*c*, *a* ≠ 0

•*a2*(*f*(*x*))2 –*b2*(*g* (*y*))2, *a* ≠ 0, *b* ≠ 0

where *a*, *b* and *c* are rational numbers.

**Factoring Quadratic Expressions**

To factor a trinomial of the form *ax2* + *bx* + *c*, where *a* ≠ 0,

Step 1)factor out common factors, if possible.

For example,

4*x2* - 2*x* - 12 =

 =

 =

 =

You can factor perfect square trinomials of the forms (*ax*)2 + 2*abx* + *b*2

and (*ax*)2 - 2*abx* + *b*2 into (*ax* + *b*)2 and (*ax* - *b*)2, respectively.

For example,

4*x*2 + 12*x* + 9 = 9*x*2 - 24*x* + 16 =

 = =

You can factor a difference of squares, (*ax*)2 - (*by*)2, into (*ax* - *by*)(*ax* + *by*).

For example,

$$\frac{4}{9}x^{2}-16y^{2}=$$

**Factoring Polynomials Having a Quadratic Pattern**

You can extend the patterns established for factoring trinomials and a difference of squares to factor polynomials in quadratic form.

You can factor a polynomial of the form *a*(*P*)2 + *b*(*P*) + *c*, where *P* is any expression, as follows:

• Treat the expression *P* as a single variable, say *r*, by letting *r* = *P*.

• Factor as you have done before.

• Replace the substituted variable *r* with the expression *P*.

• Simplify the expression.

For example, in 3(*x* + 2)2 - 13(*x* + 2) + 12, substitute *r* for *x* + 2

and factor the resulting expression,

Once the expression in *r* is factored, you can substitute *x* + 2 back in for *r*.

You can factor a polynomial in the form of a difference of squares, as *P 2* – *Q2* = (*P* - *Q*)(*P* + *Q*) where *P* and *Q* are any expressions.

For example,

(3*x* + 1)2 - (2*x* - 3)2 =

 =

 =

Example 1

Factor.

**a)** 2*x*2 **–** 2*x* **–** 12 **b)**  ¼ *x*2 **–** *x* **–** 3 **c)** 9*x*2 **–** 0.64*y*2

Example 1: Your Turn

Factor.

**a)** 3*x*2 + 3*x* **–** 6 **b) ½** *x*2 **–** *x* **–** 4 **c)** 0.49*j*2 **–** 36*k*2

Example 2: **Factor Polynomials of Quadratic Form**

Factor each polynomial.

**a)** 12(*x* + 2)2 + 24(*x* + 2) + 9 **b)** 9(2*t* + 1)2 **–** 4(*s* **–** 2)2

Example 2: Your Turn

Factor each polynomial.

**a)** *–*2(*n* + 3)2 + 12(*n* + 3) + 14 **b)** 4(*x –* 2)2 *–* 0.25(*y –* 4)2

**Solving Quadratic Equations by Factoring**

Some quadratic equations that have real-number solutions can be factored easily.

The *zero product property* states that \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

This means that if *de* = 0, then at least one of *d* and *e* is 0.

The roots of a quadratic equation occur when the \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

To solve a quadratic equation of the form *ax2* + *bx* + *c* = 0, *a* ≠ 0

Step 1) Factor the expression

Step 2) Set either factor equal to zero. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

For example, rewrite the quadratic equation 3*x2* - 2*x* - 5 = 0 in

factored form.

3*x2* - 2*x* - 5 = 0

The roots are \_\_\_\_\_\_\_\_\_ and \_\_\_\_\_\_.

**Example 3: Solve Quadratic Equations by Factoring**

Determine the roots of each quadratic equation. Verify your solutions.

**a)** *x*2 + 6*x* + 9 = 0 **b)** *x*2 + 4*x* **–** 21 = 0 **c)** 2*x*2 **–** 9*x* **–** 5 = 0

Example 3: Your Turn

Determine the roots of each quadratic equation.

**a)** *x*2 – 10*x* + 25 = 0 **b)** *x*2 – 16 = 0 **c)** 3*x*2 – 2*x* – 8 = 0

EXample 4: Applying Quadratic Equations

A waterslide ends with the slider dropping into a deep pool of water. The path of the slider after leaving the lower end of the slide can be approximated by the quadratic function

 $h(d)=-\frac{1}{6}d^{2}-\frac{1}{6}d=-2$ where *h* is the height above the surface of the pool and *d* is the horizontal distance the slider travels from the lower end of the slide, both in feet.

What is the horizontal distance the slider travels before dropping into the pool after leaving the lower end of the slide?

Example 5: Writing and Solving Quadratic Equations

The area of a rectangular Ping-Pong table is 45 ft2. The length is 4 ft more than the width. What are the dimensions of the table?

Assignment: Pg 229-233 #1, 2a,c, 3a, 4a–c, 5a,b 7a,c,d, 9a,b,d, 11, 30, 32