Lesson 3 - Rational Exponents

Algebra and Number

2. It is expected that students will:

- Demonstrate an understanding of powers with integral and rational exponents.
- applying the exponent laws to expressions using rational numbers or variables as bases and rational exponents

lesson 3

Try This

Complete each table. Use a calculator to complete the second column for each table.

x	$x^{\frac{1}{2}}$
1	$1^{\frac{1}{2}} =$
4	$4^{\frac{1}{2}} =$
9	
16	
25	

x	$x^{\frac{1}{3}}$
1	
8	
27	
64	
125	

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x	$x^{\frac{1}{2}}$
1	$1^{\frac{1}{2}} =$
4	$4^{\frac{1}{2}} =$
9	
16	
25	

What do you notice about the numbers in the first column?

Compare the numbers in the first and second columns. What conclusions can you make?

What do you think the exponent ½ means?

What do you think mean? Explain your reasoning.

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x	$x^{\frac{1}{3}}$
1	
8	
27	
64	
125	

What do you notice about the numbers in the first column?

Compare the numbers in the first and second columns. What conclusions can you make?

What do you think the exponent means?

What do you think and mean?

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Rational Exponents

Review

We know what positive exponents like 2^3 or 3^4 mean and we should know wh[illegible]*gative exponents* mean. For example:

$$2^{-3} = \frac{1}{2^3} \quad \text{and} \quad 3^{-4} = \frac{1}{3^4}$$

What, however, do we get when we want to find a fractional (rational) exponent like:

$$9^{\frac{1}{2}} = \sqrt{9}$$
$$= 3$$

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To narrow down our search, let us look at some things that we should already know. For example, $9^0 = 1$ and $9^1 = 9$. It stands to reason that, because:

$9^{1/2}$

$$0 < \frac{1}{2} < 1, \quad \text{then}$$

$$9^0 < 9^{\frac{1}{2}} < 9^1$$

$$1 < 9^{\frac{1}{2}} < 9$$

Therefore, without knowing the actual value of $9^{1/2}$ we know that it should be between 1 and 9

When we use this type of explanation to create a general rule used for all numbers it is known as **inductive reasoning**

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We know from our *Product of Powers* exponent law that:

$$a^m \times a^n = a^{(m+n)}$$

If we choose to multiply $9^{½}$ by itself we get:

$$9^{\frac{1}{2}} \times 9^{\frac{1}{2}} = 9^{\left(\frac{1}{2}+\frac{1}{2}\right)} \quad or \quad 9^1$$

But what number, multiplied by itself, will equal 9? That is, if:

$$9 = 9^{\frac{1}{2}} \times 9^{\frac{1}{2}}$$

Could we also write it like so?

$$9 = w \times w$$

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Another way to demonstrate rational exponents is to use the *Power of a Power* law

For example, the number 9 can be written as

$$\left(9^{\frac{1}{2}}\right)^2$$

because $\left(9^{\frac{1}{2}}\right)^2 = 9^{\left(\frac{1}{2}\times 2\right)}$

$$\frac{1}{2} \times \frac{2}{1} = \frac{2}{2}$$

And we also know that $9^{\left(\frac{1}{2}\times \frac{2}{1}\right)} = 9^{\left(\frac{2}{2}\right)}$ or 9^1

$$9^{1/2} = \sqrt[2]{9}$$

But we should also know that $9 = 3^2$

Thus, we should know that: $\left(9^{\frac{1}{2}}\right)^2 = 3^2$

If we remove the square root from both sides of the equation we will get $9^{\frac{1}{2}} = 3$

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Predict values for $4^{1/2}$, $16^{1/2}$, $36^{1/2}$, and $49^{1/2}$. Use a calculator to check your predictions

$4^{\frac{1}{2}} = 2$ $\quad 16^{\frac{1}{2}} = 4$ $\quad 36^{\frac{1}{2}} = 6$ $\quad 49^{\frac{1}{2}} = 7$

$\sqrt{4}$

Predict the value of $8^{\frac{1}{3}}$

$8^{\frac{1}{3}} = 2$ $\quad \sqrt[3]{8}$

Predict the value of $9^{-\frac{1}{2}}$

$9^{-\frac{1}{2}} = \frac{1}{3}$ $\quad \frac{1}{9^{1/2}}$

Predict the value of $(-8)^{-\frac{1}{3}}$

$(-8)^{-\frac{1}{3}} = -\frac{1}{2}$ $\quad \frac{1}{(-8)^{1/3}}$

$\frac{1}{-2} = -\frac{1}{2}$

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Summary of **Exponent Laws**

Match the names of the exponent laws with the appropriate expressions (note that a and b are rational or variable bases and that m and n are rational exponents

Product of Powers	$a^0 = 1,\ a \neq 0$
Quotient of Powers	$a^{-n} = \frac{1}{a^n},\ a \neq 0$
Power of a Power	$(ab)^m = (a^m)(b^m)$
Power of a Product	$(a^m)(a^n) = a^{m+n}$
Power of a Quotient	$\frac{a^m}{a^n} = a^{m-n}$
Zero Exponent	$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n},\ b \neq 0$
Negative Exponent	$(a^m)^n = a^{mn}$

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Example 1 - Simplifying Expressions with Rational Exponents

Simplify each of the following

$a)\ 27^{\frac{1}{3}}$ $\quad b)\ (-1000)^{\frac{1}{3}}$ $\quad c)\ 27^{\frac{2}{3}}$ $\quad d)\left(-\frac{1}{27}\right)^{-\frac{2}{3}}$

$\left(\frac{-27}{1}\right)^{2/3}$

$(-27)^{2/3}$

$\sqrt[3]{-27^2}$

$(-3)^2$

9

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Example 1 - Simplifying Expressions with Rational Exponents

Simplify each of the following

Solution

$a)\ 27^{\frac{1}{3}} = 3$

$b)\ (-1000)^{\frac{1}{3}} = -10$

$c)\ 27^{-\frac{2}{3}} = \frac{1}{9}$

$$d)\ \left(-\frac{1}{27}\right)^{-\frac{2}{3}} = (-27)^{\frac{2}{3}}$$
$$= (729)^{\frac{1}{3}}$$
$$= 9$$

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Example 2 - Multiplying or Dividing Powers with the Same Base

a) $\left(6^{\frac{1}{4}}\right)\left(6^{\frac{7}{4}}\right)$ b) $(n)^3\left(n^{-\frac{1}{5}}\right)$ c) $d^{\frac{3}{2}} \div d^{\frac{1}{2}}$ d) $\dfrac{3^{-\frac{2}{3}}}{3^{-\frac{4}{3}}}$

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Example 2 - Multiplying or Dividing Powers with the Same Base

a) $\left(6^{\frac{1}{4}}\right)\left(6^{\frac{7}{4}}\right)$ b) $(n)^3\left(n^{-\frac{1}{5}}\right)$ c) $d^{\frac{3}{2}} \div d^{\frac{1}{2}}$ d) $\dfrac{3^{-\frac{2}{3}}}{3^{-\frac{4}{3}}}$

a) $6^{\left(\frac{1}{4}+\frac{7}{4}\right)}$

$6^{\frac{8}{4}}$

6^2 *or* 36

b) $n^{\left(3+\frac{-1}{5}\right)}$

$n^{\left(\frac{15}{5}-\frac{1}{5}\right)}$

$n^{\frac{14}{5}}$

c) $d^{\frac{3}{2}-\frac{1}{2}}$

$d^{\frac{2}{2}}$

d^1 or d

d) $3^{-\frac{2}{3}-\frac{-4}{3}}$

$3^{-\frac{2}{3}+\frac{4}{3}}$

$3^{\frac{2}{3}}$

$\dfrac{3\times 5}{1\times 5}+\dfrac{-1}{5}$

$\dfrac{15}{5}+\dfrac{-1}{5}=\dfrac{14}{5}$

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