Chapter 4

4.4 The Quadratic Formula

Focus On ...

- Developing the Quadratic Formula
- solving quadratic equations using the quadratic formula
- Using the discriminant to determine the nature of the roots of a quadratic equation
- Selecting an appropriate method for solving quadratic equations
- Solving problems involving quadratic equations

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4.4

Investigate the Quadratic Formula

By completing the square you can develop a formula that allows you to solve any quadratic equation in standard form

1. Describe the steps in the following example of the quadratic formula.

Quadratic Formula:
A formula for determining the roo
of a quadratic equation of the for

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$
\begin{aligned}
2x^2 + 7x + 1 &= 0\\
x^2 + \frac{7}{2}x + \frac{1}{2} &= 0\\
x^2 + \frac{7}{2}x &= -\frac{1}{2}\\
x^2 + \frac{7}{2}x + \left(\frac{7}{4}\right)^2 &= -\frac{1}{2} + \left(\frac{7}{4}\right)^2\\
\left(x + \frac{7}{4}\right)^2 &= -\frac{8}{16} + \frac{49}{16}\\
\left(x + \frac{7}{4}\right)^2 &= \frac{41}{16}\\
x + \frac{7}{4} &= \pm\sqrt{\frac{41}{16}}\\
x &= -\frac{7}{4} \pm \frac{\sqrt{41}}{4}\\
x &= \frac{-7 \pm \sqrt{41}}{4}
\end{aligned}
$$

2. Repeat the steps using the general quadratic equation in standard form $ax^2 + bx + c = 0$.

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Reflect and Respond

3. a) Will the quadratic formula work for any quadratic equation written in any form?

b) When do you think it is appropriate to use the quadratic formula to solve a quadratic equation?

c) When is it appropriate to use a different method, such as graphing the corresponding function, factoring, determining the square root, or completing the square? Explain.

4. What is the maximum number of roots the quadratic formula will give? How do you know this?

5. Describe the conditions for *a*, *b*, and *c* that are necessary for the quadratic formula, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, to result in only one possible root.

6. Is there a condition relating *a*, *b*, and *c* that will result in no real solution to a quadratic equation? Explain.

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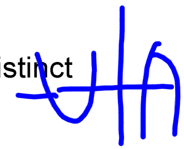
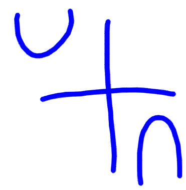
You can determine the nature of the roots for a quadratic equation by the value of the discriminant. The discriminant is the expression b^2-4ac located under the radical sign in the quadratic formula.

• When the value of the discriminant is positive, >0, there are 2 distinct real roots.



• When the value of the discriminant is zero, $=0$, there is 1 distinct real root, or two equal real roots.

• When the value of the discriminant is negative, <0, there are no **real roots.**



You can see that this is true by testing the three different types of values of the discriminant in the quadratic formula.

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Example 1:

Use the Discriminant to Determine the Nature of the Roots

Use the discriminant to determine the nature of the roots for each quadratic equation. Check by graphing.

a) $-2x^2 + 3x + 8 = 0$

b) $3x^2 - 5x = -9$

$+9 \quad +9$

c) $\frac{1}{4}x^2 - 3x + 9 = 0$

$b^2 - 4ac$

$3^2 - 4(-2)(8$

$9 + 64 = 73$

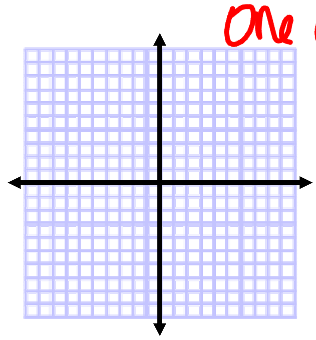
2 real roots

$(-3)^2 - 4(1/4)(9)$

$9 - 9 = 0$

one real root.

a)



$3x^2 - 5x + 9 = 0$

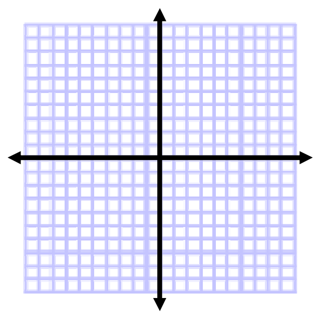
$(-5)^2 - 4(3)(9)$

b)

$25 - 108$

-83

zero real roots.



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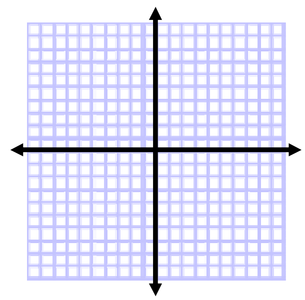
c)



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You can solve quadratic equations of the form $ax^2 + bx + c = 0$, $a \neq 0$, using the quadratic formula, $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

For example, in the quadratic equation $3x^2 + 5x - 2 = 0$, $a = 3$, $b = 5$, and $c = -2$.

Substitute these values into the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-5 \pm \sqrt{5^2 - 4(3)(-2)}}{2(3)}$$

$$x = \frac{-5 \pm \sqrt{25 + 24}}{6}$$

$$x = \frac{-5 \pm \sqrt{49}}{6}$$

$$x = \frac{-5 \pm 7}{6}$$

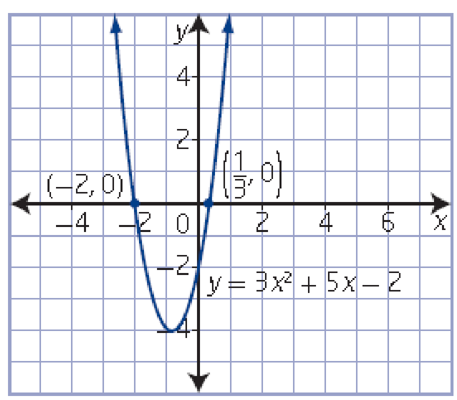
Determine the two roots.

$$x = \frac{-5 + 7}{6} \quad \text{or} \quad x = \frac{-5 - 7}{6}$$

$$x = \frac{1}{3} \qquad\qquad x = \frac{-12}{6}$$

$$x = -2$$

The roots are $\frac{1}{3}$ and -2.



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Example 2:

Select a Strategy to Solve a Quadratic Equation

a) Solve $6x^2 - 14x + 8 = 0$ by

i) graphing the corresponding function

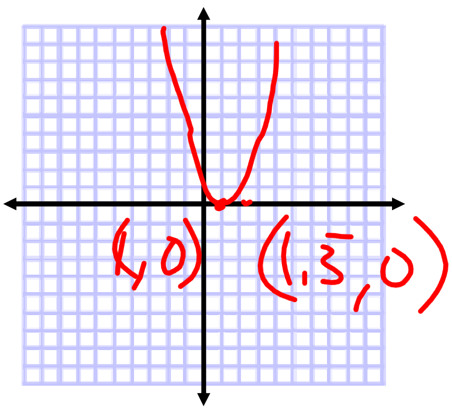
ii) factoring the equation

~~**iii)** completing the square~~

iv) using the quadratic formula

b) Which strategy do you prefer? Justify your reasoning.

a)



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ii. Factoring

$$6x^2-14x+8=0$$

$$2(3x^2-7x+4)=0$$

$$2(3x-4)(x-1)=0$$

12

-3 -4

-7

iii. ~~Completing the Square~~

$$3x-4=0$$
$$+4 \quad +4$$
$$\frac{3x}{3}=\frac{4}{3}$$
$$x=4/3$$

$$x-1=0$$
$$+1 \quad +1$$
$$x=1$$

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iv. Quadratic Formula

$6x^2 - 14x + 8 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-14) \pm \sqrt{(-14)^2 - 4(6)(8)}}{2(6)}$$

Which do you prefer? Why? When would you use each method?

$$= \frac{14 \pm \sqrt{196 - 192}}{12}$$

$$= \frac{14 \pm \sqrt{4}}{12} = \frac{14 \pm 2}{12}$$

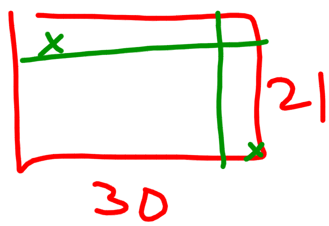
$$\frac{14+2}{12} = \frac{16}{12} = \frac{4}{3}$$

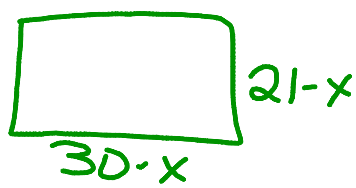
$$\frac{14-2}{12} = \frac{12}{12} = 1$$

4.4

Example 3: Applying the Quadratic Formula

A picture measures 30 cm by 21 cm. You crop the picture by removing strips of the same width from the top and one side of the picture. This reduces the area to 40% of the original area. Determine the width of the removed strips.





$A = lw$
$= 30 \times 21$
$= 630$

$A = 40\%$ of 630
$A = 0.4 \times 630$
$A = 252$

$A = lw$
$252 = (30 - x)(21 - x)$
$252 = 630 - 30x - 21x + x^2$
$-252 \quad -252$
$0 = x^2 - 51x + 378$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \dfrac{-(-51) \pm \sqrt{(51)^2 - 4(1)(378)}}{2(1)}$

$x = \dfrac{51 \pm \sqrt{2601 - 1512}}{2}$

$x = \dfrac{51 \pm \sqrt{1089}}{2}$

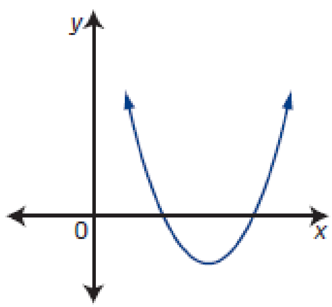
$x = \dfrac{51 \pm 33}{2}$

$\dfrac{51 + 33}{2} = \dfrac{84}{2} = 42$

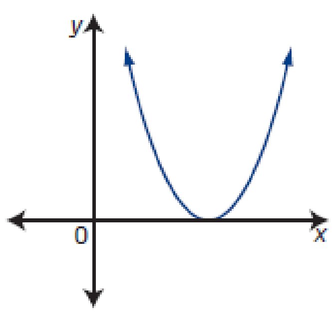
$\dfrac{51 - 33}{2} = \dfrac{18}{2} = 9$

4.4 Key Ideas

- You can solve a quadratic equation of the form $ax^2 + bx + c = 0$, $a \neq 0$, for x using the quadratic formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.
- Use the discriminant to determine the nature of the roots of a quadratic equation.
  - When $b^2 - 4ac > 0$, there are two distinct real roots. The graph of the corresponding function has two different x-intercepts.

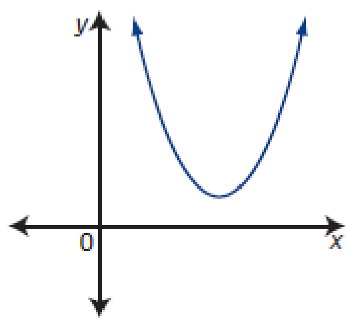


  - When $b^2 - 4ac = 0$, there is one distinct real root, or two equal real roots. The graph of the corresponding function has one x-intercept.



4.4 Key Ideas

- When $b^2 - 4ac < 0$, there are no real roots. The graph of the corresponding function has no x-intercepts.



- You can solve quadratic equations in a variety of ways. You may prefer some methods over others depending on the circumstances.

Assignment: Pg 254-257 #'s 1,2&5 (b,d,f), 9,10,12,15, 22,23

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