4.4 Number Systems and Irrational Numbers:

Demonstrate an understanding of irrational numbers by:

- representing, identifying and simplifying irrational numbers
- ordering irrational numbers.

Work in groups to answer the next three questions.

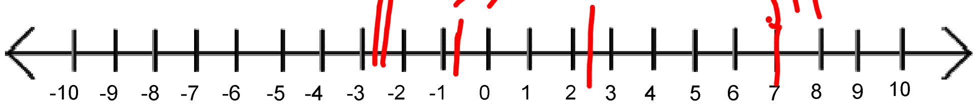
1. Given a table with a selection of rational and irrational numbers, what generalizations can you make about rational and irrational numbers based on the values provided below.

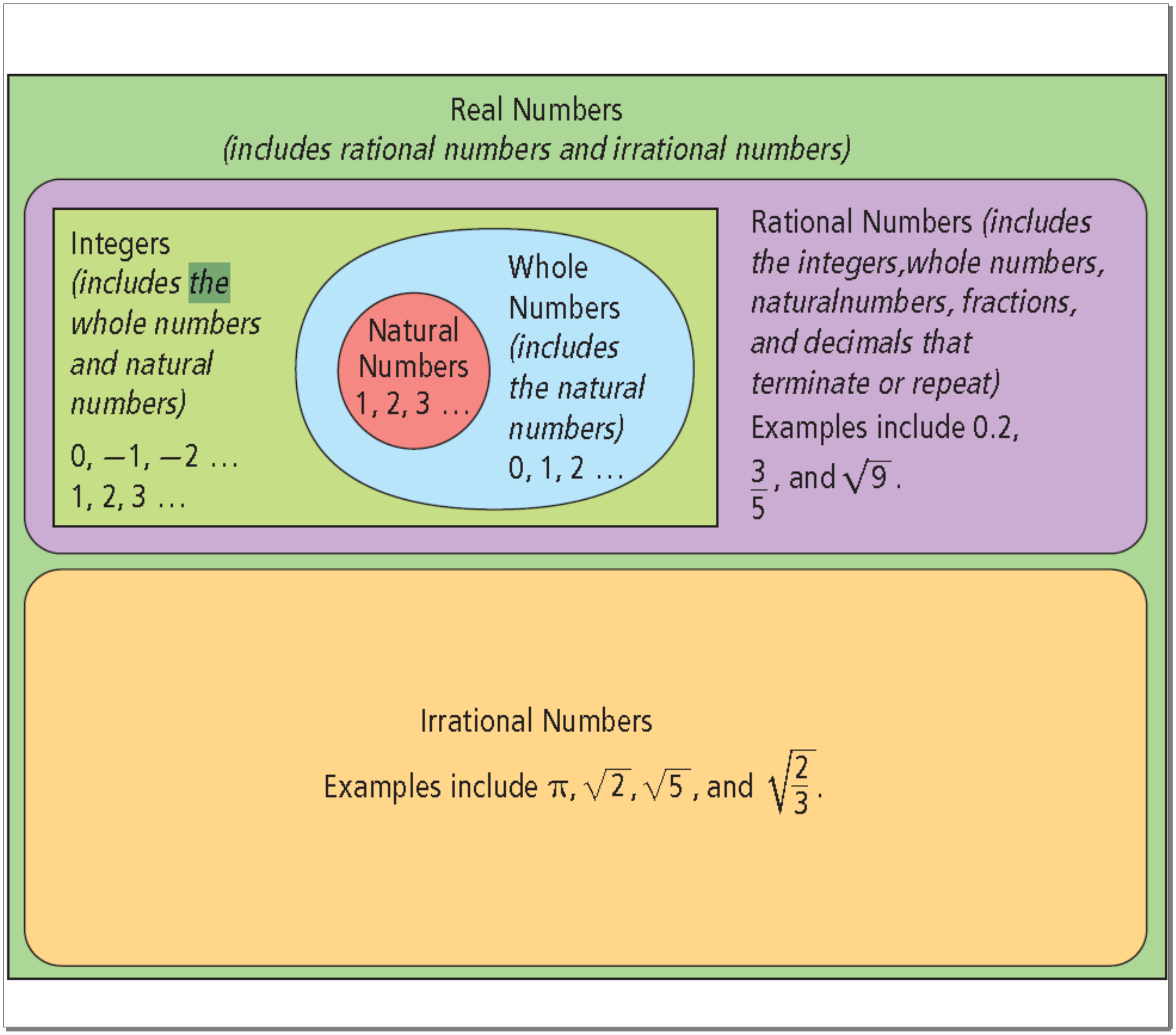
Rational	Irrational Numbers
$-\sqrt{4}$ $\sqrt[3]{8}$ $\frac{3}{4}$ $-\frac{7}{3}$ $\sqrt{64}$	$-\sqrt{8}$ $\sqrt{27}$ $\sqrt[3]{-24}$ π
$\sqrt{0.36}$ $\sqrt{\frac{16}{25}}$ 5^{-2}	$6^{-\frac{1}{2}}$ $\sqrt{0.6}$

2. Define real numbers, integers, whole numbers and natural numbers.

3. Given the following numbers: $\sqrt{5}$, $\frac{-2}{3}$, $-\sqrt[3]{24}$, $-\sqrt{7}$, $\sqrt{49}$.

- Classify each number as being rational or irrational.
- Order the numbers from lowest to highest.
- Locate them on a number line.




Real Numbers
(includes rational numbers and irrational numbers)
Integers
(includes the whole numbers and natural numbers)
0, −1, −2 …
1, 2, 3 …
Natural Numbers
1, 2, 3 …
Whole Numbers
(includes the natural numbers)
0, 1, 2 …
Rational Numbers (includes the integers,whole numbers, naturalnumbers, fractions, and decimals that terminate or repeat)
Examples include 0.2, $\frac{3}{5}$, and $\sqrt{9}$.
Irrational Numbers
Examples include π, $\sqrt{2}$, $\sqrt{5}$, and $\sqrt{\frac{2}{3}}$.

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Just as with fractions, equivalent expressions for any number have the same value.

$\sqrt{16\times 9}$ is equivalent to $\sqrt{16}\bullet\sqrt{9}$ because:

$$\sqrt{16\times 9}$$
$$=\sqrt{144}$$
$$=12$$

$$\sqrt{16}\bullet\sqrt{9}$$
$$=4\bullet 3$$
$$=12$$

Multiplication Property of Radicals

$$\sqrt[n]{ab}=\sqrt[n]{a}\times\sqrt[n]{b},$$

where n is a natural number, and a and b are real numbers

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Chapter 4

Irrational Numbers

A power with a fractional exponent ($x^{\frac{1}{n}}$) can be written as a Drag text here.

Drag text here → n

← Drag text here root symbol

$$\sqrt[n]{x}$$

↑

Drag text here

Chapter 4

Irrational Numbers

Sort the following radicals.

Mixed radicals	Entire radicals
$\frac{1}{2}\sqrt{7}$	$\sqrt[3]{\frac{4}{5}}$
$6\sqrt[3]{4}$	$\sqrt[3]{2^6}$
$3\sqrt{5}$	$\sqrt{42}$
$0.2\sqrt{10}$	$\sqrt{70}$

Click here for the answer

Example 3 Convert Mixed Radicals to Entire Radicals

Express each **mixed radical** as an equivalent **entire radical**

a) $5\sqrt{11}$

b) $2\sqrt[3]{5}$

c) $1.5\sqrt[3]{6}$

a) $5\sqrt[2]{11}$: $= 16.6$

$\sqrt{11(5)^2}$

$\sqrt{11(25)}$

$\sqrt{275}$ $= 16.6$

b) $2\sqrt[3]{5}$

$\sqrt[3]{5(2)^3}$

$\sqrt[3]{5(8)}$

$\sqrt[3]{40}$

c) $1.5\sqrt[3]{6}$ $= 2.7$

$\sqrt[3]{6(1.5)^3}$

$\sqrt[3]{6(3.375)}$

$\sqrt[3]{20.25}$ $= 2.7$

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Example 3 Convert Mixed Radicals to Entire Radicals
Express each **mixed radical** as an equivalent **entire radical**

Solutions

a) $$5\sqrt{11} = \sqrt{5^2}\sqrt{11} = \sqrt{(5^2)(11)} = \sqrt{(25)(11)} = \sqrt{275}$$

b) $$2\sqrt[3]{5} = \sqrt[3]{(2^3)}\sqrt[3]{5} = \sqrt[3]{(2^3)(5)} = \sqrt[3]{(8)(5)} = \sqrt[3]{40}$$

c) $$1.5\sqrt[3]{6} = \sqrt[3]{(1.5^3)}\sqrt[3]{6} = \sqrt[3]{(1.5^3)(6)} = \sqrt[3]{(3.375)(6)} = \sqrt[3]{20.25}$$

Your Turn

Express each radical as a power.

a) $\sqrt{125}$ b) $\sqrt[3]{y^5}$ c) $\sqrt[n]{27^2}$

$\frac{1}{3}\sqrt{10} = \sqrt{10\left(\frac{1}{3}\right)^2}$

$\sqrt{\frac{10}{1}\left(\frac{1}{9}\right)} = \sqrt{\frac{10}{9}}$

Convert each mixed radical to an equivalent entire radical.

a) $9\sqrt[3]{4}$ b) $4.2\sqrt[2]{18}$ c) $\frac{1}{2}\sqrt{10}$

$\sqrt[3]{4 \cdot 9^3}$ $\sqrt{18(4.2)^2}$ $0.5\sqrt{10}$

$\sqrt[3]{4(729)}$ $\sqrt{18(17.64)}$ $\sqrt{10(0.5)^2}$ $\sqrt{2.5}$

$\sqrt[3]{2916}$ $\sqrt{317.52}$ $\sqrt{10(0.25)}$

Recall the Perfect Square Numbers are:

Recall the perfect cube numbers are:

Example 4 Convert Entire Radicals to Mixed Radicals

Express each entire radical as an equivalent mixed radical.

a) $\sqrt{27}$ *b*) $\sqrt{50}$ *c*) $\sqrt{48}$ *d*) $\sqrt[3]{54}$

Example 4 Convert Entire Radicals to Mixed Radicals
Express each entire radical as an equivalent mixed radical.

$a)\ \sqrt{27}$ $b)\ \sqrt{50}$ $c)\ \sqrt{48}$ $d)\ \sqrt[3]{54}$

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Example 4 Convert Entire Radicals to Mixed Radicals

Express each entire radical as an equivalent mixed radical.

Solutions

a) $\sqrt{27} = \sqrt{(9)(3)}$
$= \sqrt{9}\sqrt{3}$
$= 3\sqrt{3}$

b) $\sqrt{50} = \sqrt{(25)(2)}$
$= \sqrt{25}\sqrt{2}$
$= 5\sqrt{2}$

c) $\sqrt{48} = \sqrt{(16)(3)}$
$= \sqrt{16}\sqrt{3}$
$= 4\sqrt{3}$

d) $\sqrt[3]{54} = \sqrt[3]{(27)(2)}$
$= \sqrt[3]{27}\sqrt[3]{2}$
$= 3\sqrt[3]{2}$

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Your Turn

Convert each entire radical to an equivalent mixed radical.

a) $\sqrt{40}$ b) $\sqrt{108}$ c) $\sqrt[3]{32}$