6.4 Functions

Focus on ...
- sorting relations into functions and non-functions
- using notation specifically designed for functions
- graphing linear functions

Recall: A relation is an ___association___ between two quantities which can be written in words, as an equation, as ordered pairs, as a table of values or as a graph.

Functions → Relations that always have ___only one output value___ for each ___input value___.
→ Relations that have a unique ___range___ value for every ___domain___ value.

Investigate Functions

Study the following relations. They are categorized as functions and non-functions.

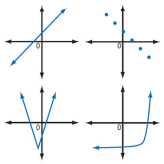
These 8 relations are functions.

x	y
-5	10
6	15
7	20

x	y
11	3
21	3
31	3

{(−2, −5), (0, 4), (2, 13), (4, 22)}

{(10, 10), (12, 10), (14, 12), (16, 12)}



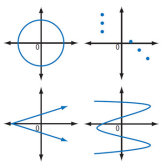
These 8 relations are not functions.

x	y
6	10
6	15
7	20

x	y
3	11
3	21
3	31

{(10, 10), (12, 10), (12, 14), (12, 16)}

{(7, 5), (7, 8), (9, 11), (11, 14)}



With your partner discuss how you can tell:
- Which tables of values are functions and which are not.

  _____
- Which ordered pairs represent functions and which do not.

  _____
- Which graphs represent function and which do not.

  _____

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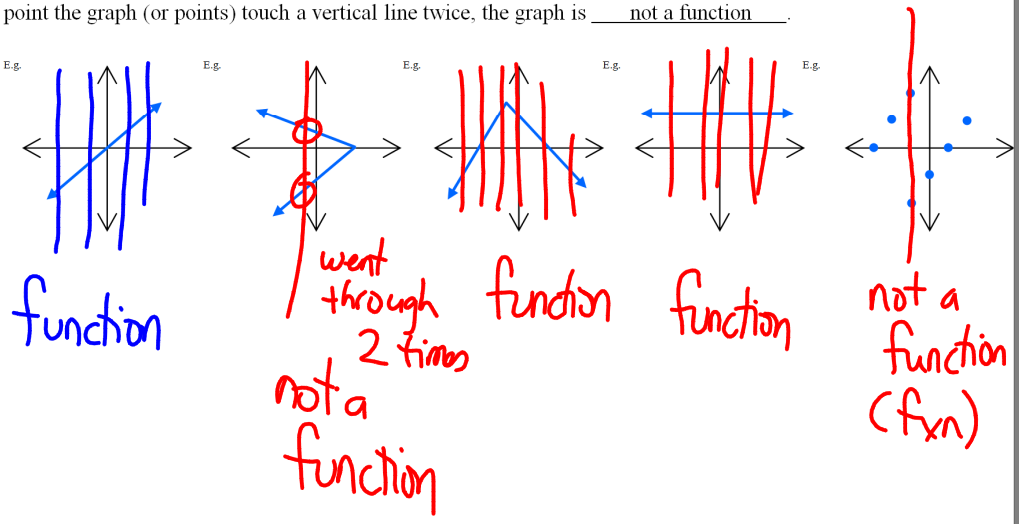
The Vertical Line Test

Functions are a one-to-one relation. With this idea in mind, if we think of each independent (input) value as the x values, then we are on the lookout for x values which map to two y values.

One method to identify graphs as functions or just relations is to ___scan___ across with vertical lines. If at any point the graph (or points) touch a vertical line twice, the graph is ___not a function___.



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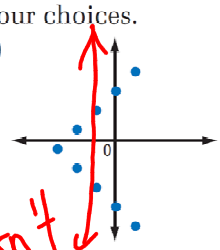
Your Turn

Which of the following relations are functions? Explain your choices.

a)



doesn't pass vertical line test

b) $\{(-2, 1), (0, 0), (2, 1), (5, 1)\}$

function → only one of each x-value.

c)

x	y
1	3
2	3
3	4
4	4
5	4

function

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Function Notation

Recall: Relations represent an ____association____ between two quantities.

E.g. The more sunlight there is, the warmer it is.

What two quantities are described in this example?

____sunlight____ and ____temperature____.

Which one depends on the other?

____temperature____ depends upon ____sunlight____.

Mathematicians say that ____temperature____ is a function of ____sunlight____.

There is a notation for writing functions:	Temperature is a function of sunlight.
Variable to represent independent value	*s*
Variable to represent dependent value	*t*
Notation	[redacted]
Read as	[redacted] OR [redacted]

t(s)

tofs tats

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E.g. Mark is buying a pizza. If he's the only person it costs $20. If he has a buddy it's $10 each. If he has two buddies it's $6.67 each. If he has three buddies it's $5.00 each and so on.

What two quantities are described in this example?

_____number of people_____ and _____cost_____.

Which one depends on the other?

_____cost_____ depends upon _____number of people_____.

Mathematicians say that _____cost_____ is a function of _____number of people_____.

There is a notation for writing functions:	Cost is a function of the number of people.
Variable to represent independent value	n
Variable to represent dependent value	C
Notation	[redacted]
Read as	[redacted] OR [redacted]

C(n)
Cofn or Catn

The expressions $t(s)$ and $C(n)$ lack meaning. The rule that defines the association between the variables needs to be included to have function notation.

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E.g. Consider the number of shoes in the room. The number of shoes is a value that is dependent upon another quantity, _____the number of people in the room_____. What would be a rule to determine the number of shoes in the room?

State the dependent and independent variables

# of shoes (S) ↘ # of people. (P)

Write the relationship as a formula

$S = 2p$

State the variables as a function

P

S(P)

Write the relationship in function notation

$S(p) = 2p$

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Consider the amount of money Bob gets paid. This money is called income.

Hours Worked	3	4	5	6	7	8
Income	$27.60	$36.80	$46.00	$55.20	$64.40	$73.60

$\frac{27.60}{3} = \$9.20$

State the dependent and independent variables

income (i) # of hours (h)

Write the relationship as a formula

$i = 9.20h$

p. 311 ques 1-6

State the variables as a function

h

i(h)

Write the relationship in function notation

$i(h) = 9.20h$

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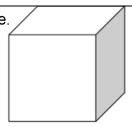
Consider the surface area of a cube.



State the dependent and independent variables

Write the relationship as a formula

State the variables as a function

Write the relationship in function notation

One or the convenient ways to think of functional notation is that it takes the place of the dependent variable. This means we can graph functions the same way we graphed any other relation. *f(x)* is another way of writing *y*.

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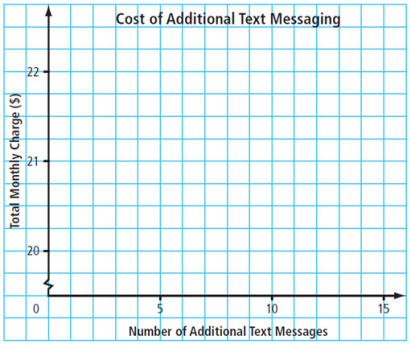
Skye has a cell phone plan for a monthly fee of $20 plus 15¢ for each text message to or from a number not on a list of favourites. Skye's monthly bill can be modelled by the relation $C = 0.15n + 20$, where C is the total charge, in dollars, and n is the number of additional text messages.

a) Write the relation in function notation.

b) Make a table of values. Graph the function if Skye sends up to four additional text messages.

n	C(n)
0	
5	
10	
13	
15	
150	



c) If Skye's cell phone bill for a certain month is $22.25, how many additional text message charges are there?

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The most common variable used to represent a function is f but other variables can also be used.

For example, given $f(x) = 3x-2$,

Find $f(5)$

$f(-3)$

Determine x, if $f(x)=13$

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Your Turn

Given $f(x) = 5x + 7$

a) Determine

$f(-3)$ $f(9)$ $f(a)$

b) $f(x) = -18$, determine x

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Assignment: Page 311 #1-7, 8, 10abc, 14, 15, 17, 18

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Check Your Understanding

Practise

1. Determine whether each relation is a function or is not a function. Give a reason for your answer.

a) $(-1, 2), (0, 1), (1, 2), (2, 5)$

b) $(3, 12), (4, 12), (5, 14), (6, 14)$

c) $(1, 2), (2, 3), (3, 4), (4, 5), (5, 6)$

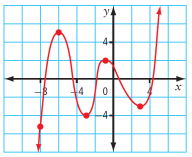
d)

x	y
0	0
1	−1
1	1
4	−2
4	2

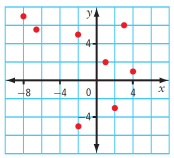
e)

Name	Age
Naomi	14
Wam	15
Brigid	14
Sharon	16
Arvind	15

f)



g)



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2. The formula for the surface area, A, of a sphere with radius r is $A = 4\pi r^2$. Write this formula using function notation.

3. The cost to have artwork printed on T-shirts is given by the function $C(n) = 3n + 50$, where n is the number of shirts and C is the cost, in dollars. Write this function as a formula in two variables.

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5. If $h(x) = \frac{2}{3}x + 1$, determine

a) $h(9)$ **b)** $h(-3)$ **c)** x if $h(x) = -7$

6. Consider the function $p(x) = -4x + 2$.

a) What is the value of $p(0)$?

b) Determine x so that $p(x) = -2$.

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7. Make a table of values and graph each function.

a) $g(x) = -3x + 5$ for the domain $\{-3, -2, -1, 0, 1, 2, 3\}$

b) $h(x) = \frac{x}{2}$ for the domain $\{x \mid -10 \leq x \leq 10, x \in \mathrm{R}\}$

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Apply

8. Mike currently has \$200 and saves \$20 each week. The function $M(w) = 20w + 200$ describes his saving pattern. Ali currently has \$200 and spends \$20 each week. The function $A(w) = 200 - 20w$ describes her spending pattern.

a) What does the variable w represent in each function?

b) Explain the meaning of $M(w)$ and $A(w)$.

c) What is the value of each function when $w = 4$? Explain your answer.

d) Determine the value of w when $A(w) = 0$. Explain your answer.

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10. Weight on the moon is not the same as it is on Earth because of differences in the force of gravity. The function $m(E) = \frac{E}{6}$ can be used to approximate your weight, m, on the moon, where E represents your weight on Earth.

a) Does the function indicate that you would be heavier or lighter on the moon than on Earth? Explain.

b) If a person weighs 80 kg on Earth, how much would the person weigh on the moon?

c) How much would you weigh on the moon?

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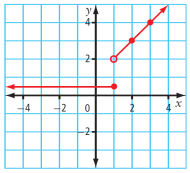
14. **a)** Does the graph represent a function? Explain.

b) What is the value of $f(-4)$? $f(1)$? $f(3)$? $f(5)$?



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15. The input for a function can be another function. If $h(x) = 2x - 5$, determine a simplified expression for each of the following.

a) $h(4x)$ **b)** $h(2x + 3)$ **c)** $h\left(\frac{x}{2} - 1\right)$

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17. Explain the difference between $f(2)$ and $f(x) = 2$.

18. Jean-Marie has never seen function notation. When he sees a question that asks him to determine the value of $f(x + 2)$, he gives his answer as $fx + 2f$.

a) How does Jean-Marie interpret the question?

b) Explain the meaning of this question to Jean-Marie in the context of functions.

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