Lesson 7.3 Slope-Point Form of an Equation of a Line
6. Relate linear relations expressed in:

- slope-intercept form $(y=m x+b)$
- general form $(A x+B y+C=0)$
- slope-point form $\left(y-y_{1}=m\left(x-x_{1}\right)\right)$ to their graphs

7. Determine the equation of a linear relation, given:

- a graph
- a point and the slope
- two points
- a point and the equation of a parallel or perpendicular line to solve problems

The slope of a non-vertical line can be determined using $m=\frac{\Delta y}{\Delta x}$
If $\left(x_{1}, y_{1}\right)$ is one point on the line, then $(x, y)$ could represent any other
point on the line. Substitute the coordinates of these two points into the slope formula, $m=\frac{y_{2}-y_{1}}{x_{2}-x_{1}}$

The slope of the line could be written as $m=\frac{y-y_{1}}{x-x_{1}}$


$$
m=\frac{y-y_{1}}{x-x_{1}}
$$

Multiplying both sides of the above equation by $\left(x-x_{1}\right)$ gives

$$
\begin{aligned}
& \left(x-x_{1}\right) m=\frac{y-y_{1}}{x-x_{1}}\left(x-x_{1}\right) \\
& \left(x-x_{1}\right) m=y-y_{1}
\end{aligned}
$$

This equation is called the slope-point form of a non-vertical line through point $\left(x_{1}, y_{1}\right)$ with slope $m$.
The slope-point form is commonly written as $y-y_{1}=m\left(x-x_{1}\right)$.
It is more commonly written as $y-y_{1}=m\left(x-x_{1}\right)$

Example 1 Write the Equation of a Line Using a Point and the Slope
a) Use slope-point form to write an equation of the line through $(-2,5)$ with slope -3 .
b) Express the equation in slope-intercept form, $y=m x+b$.
c) Graph the linear relation using technology.

Solution
a) Substitute -3 for $m$ and the coordinates of the point $(-2,5)$ for $\left(x_{1}, y_{1}\right)$
$y-y_{1}=m\left(x-x_{1}\right)$
$y-(5)=-3(x-(-2))$
$y-5=-3(x+2)$
The equation in slope-point form is $y-5=-3(x+2)$.
b) To express the equation in slope-intercept form, isolate $y$.
$y-5=-3(x+2)$
$y=-3(x+2)+5$
$y=-3 x-6+5$
$y=-3 x-1$
In slope-intercept form, the equation is $y=-3 x-1$.
your Turn
a) Use slope-point form to write an equation of the line through $(3,-4)$. with slope 2. Sketch a graph of the line.
b) Express the equation in slope-intercept form, $y=m x+b$. Sketch a graph of this line.
a.



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Example 2 Determine the Equation of a Line Using Two Points
a) Use slope-point form to write an equation of the line through (3, -4) and (5, -1).
b) Sketch a graph of the line.
c) Rewrite the equation in general form, \(A x+B y+C=0\).
Solution
a) Points on the line are given. So, you need to determine the slope
Use the two given points, \((3,-4)\) and \((5,-1)\).
    \(m=\frac{y-y_{1}}{x-x_{1}} \quad x_{1} y_{1} \quad x_{2} y_{2}\)
    \(m=\frac{-1--4}{5-3}\)
    \(m=\frac{3}{2}\)
In slope-point form, substitute \(\frac{3}{2}\) for \(m\) and the coordinates of either
point \((3,-4)\) or point \((5,-1)\) for \(\left(x_{1}, y_{1}\right)\).
    Using \((3,-4)\) for \((x, y), \quad \quad\) Using \((5,-1)\) for \((x, y)\),
    \(\begin{array}{lll}y-(-4)=\frac{3}{2}(x-3) \\ y+4 & =\frac{3}{2}(x-3) \\ y+4 & =\frac{3}{2} x-\frac{9}{2}-4 \\ -4\end{array} \quad y=\frac{3}{2} x-\frac{9}{2}-\frac{8}{2} \quad y-(-1)=\frac{3}{2}(x-5) \quad y: \frac{3}{2} x-\frac{15}{2}-\frac{2}{2}\)
        Both \(y+4=\frac{3}{2}(x-3)\) and \(y+1=\frac{3}{2}(x-5)\)
        are slope-point forms of the equation of the line through \((3,-4)\) and \((5,-1)\).
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b)

C) Express $y+4=\frac{3}{2}(x-3)$ in general form.

$$
\begin{aligned}
& 2[y+1]=\not 2\left[\frac{3}{\not x}(x-5)\right] \\
& 2(y+1)=3(x-5) \\
& 2 y+2=3 x-15 \\
& 0=3 x-2 y-17
\end{aligned}
$$

The equation, in general form, for the line through $(3,-4)$ and $(5,-1)$ is

$$
3 x-2 y-17=0 .
$$

Your Turn
Use slope-point form to write an equation of the line through $(-5,2)$ and $(-2,1)$. Explain your steps. Then, write the equation in general form, $A x+B y+C=0$.

