7.4 Parallel and Perpendicular Lines

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7.4 Parallel and Perpendicular Lines

Focus on …

- identifying whether two lines are parallel, perpendicular, or neither

- writing the equation of a line using the coordinates of a point on the line and the equation of a parallel or perpendicular line

- solving problems involving parallel and perpendicular lines

Objectives

1

Definition: **Parallel** - two lines in the same ___plane___ which never ___intersect___ .
Definition: **Perpendicular** - two lines that ___intersect___ at ___right angles___ to each other.

E.g. Draw line *CD* parallel to *AB*, then draw line *EF* perpendicular to *AB*.

A

B

Definitions

Definition: **Parallel** - two lines in the same ___plane___ which never ___intersect___ .
Definition: **Perpendicular** - two lines that ___intersect___ at ___right angles___ to each other.

E.g. Draw line *CD* parallel to *AB*, then draw line *EF* perpendicular to *AB*.

A

F

C

B

D

E

Definitions

2

The next activity is not found in the notes. The results will be summarized following the activity.

Note

Drag the thumbnail of the picture to the Cartesian plane so that point A is at (0, 0) and point B is at (3, 4).
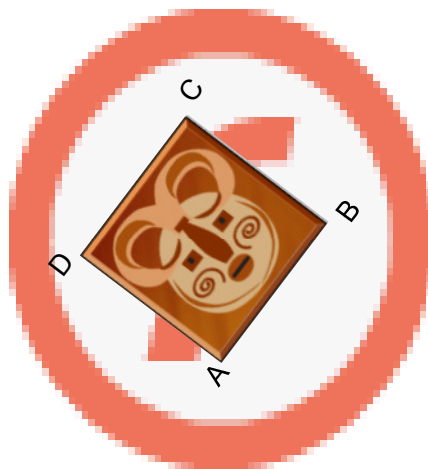
The coordinates of vertex C are

$\left(\; f \;, \; f \; \right)$

The coordinates of vertex D are

$\left(\; f \;, \; f \; \right)$

Click here for the answer

Think 1

Answer the following questions. Reveal your answers with the magnifying glass.

Which side is parallel to AB?

Determine the slope of each of the parallel sides.

How are the slopes of the
parallel sides related?

The slopes are _____ .

Pull to here

Answer

Think 2

Which side is perpendicular to AB?
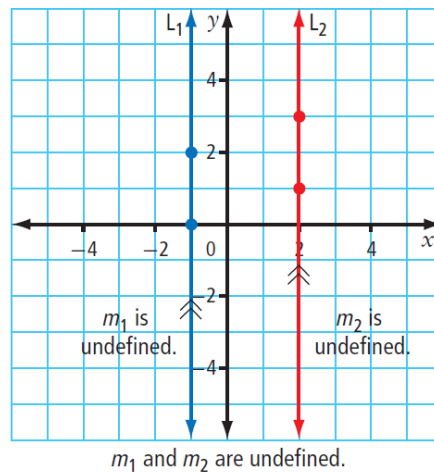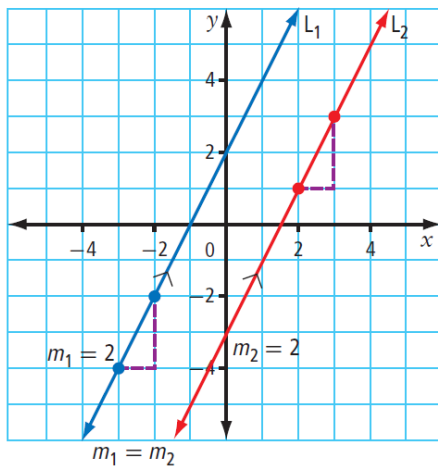
Determine the slope of each of the perpendicular sides.

How are the slopes of the
perpendicular sides related?

The slopes are _____

_____ .

Pull to here

Answer

Think 3

Link the Ideas

Parallel lines have the same slope but different intercepts. This includes horizontal lines, which have a slope of zero. Vertical lines, which have an undefined slope, are also parallel.
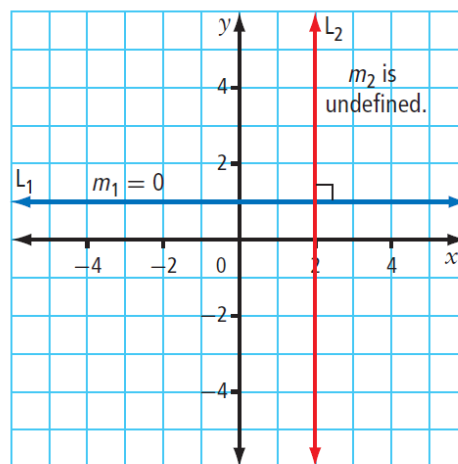

Parallel Notes

The slopes of **perpendicular lines** are negative reciprocals of each other. The product of negative reciprocals is -1. A vertical line, which has an undefined slope, and a horizontal line, which has a slope of 0, are perpendicular to each other.
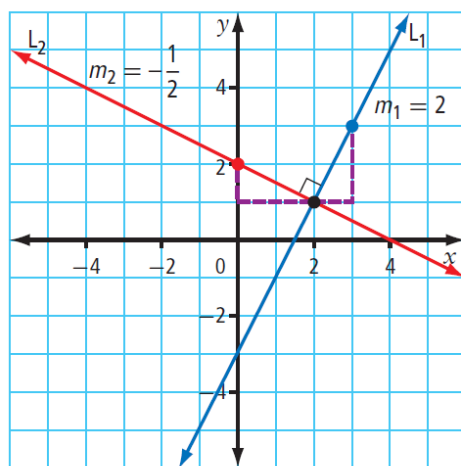
Perpendicular Notes

A possible solution on the next page.

All pairs of lines (or linear relations) fall into one of three categories:

Either their slopes are   <u>same</u>   and the lines are   <u>parallel</u>   .
Or their slopes are   <u>negative reciprocals</u>   and the lines are   <u>perpendicular</u>   .
Or slopes are not directly related so the lines are   <u>neither</u>   parallel or perpendicular.

Example 1   Identify Parallel and Perpendicular Lines

State whether the lines in each pair are parallel, perpendicular, or neither.

a) $y = 3x - 6$

$y = -\frac{1}{3}x + 4$

$Slope = 3$
$Slope = -\frac{1}{3}$ } negative reciprocal

Perpendicular.

b) $y = 4x + 3$

$y = 4x - 5$

$Slope = 4$
$diff\ y-int$
$parallel$

c) $y = 2x + 6$ ←

$6x + 3y + 3 = 0$
$-6x \quad -3$ ÷3

$\frac{3y}{3} = \frac{-6x}{3} - \frac{3}{3}$

$y = -2x - 1$ ←

$neither$

Example 1

$m_1 = 3$

$m_2 = \frac{-1}{3}$

$m_1 \perp m_2$

$m_1 = 4$

$m_2 = 4$

$m_1 \parallel m_2$

$6x + 3y + 3 = 0$

$3y = -6x - 3$

$y = -2x - 1$

$m_1 = 2$

$m_2 = -2$

Neither

Example 1

Your Turn

Determine whether the lines in each pair are parallel, perpendicular, or neither.

a) $y = \frac{1}{2}x - 7$ **b)** $y = 3x - 4$ **c)** $y = \frac{2}{5}x - 6$

$y = 2x - 7$ $y = 3x + \frac{1}{4}$ $5x + 2y = 8$

neither.   *parallel*

$-5x \qquad -5x$

$\frac{2y}{2} = \frac{-5x + 8}{2}$

$y = \frac{5}{2}x + 4$

perpendicul

Your Turn 1

Your Turn

Determine whether the lines in each pair are parallel, perpendicular, or neither.

a) $y = \frac{1}{2}x - 7$ **b)** $y = 3x - 4$ **c)** $y = \frac{2}{5}x - 6$

$y = 2x - 7$ $y = 3x + \frac{1}{4}$ $5x + 2y = 8$

$m_1 = \frac{1}{2}$ $m_1 = 3$ $\dfrac{2y}{2} = \dfrac{-5x + 8}{2}$

$m_2 = 2$ $m_2 = 3$ $y = \frac{-5}{2}x + 4$

Neither $m_1 \parallel m_2$

$m_1 = \frac{2}{5}$

$m_2 = \frac{-5}{2}$

$m_1 \perp m_2$

Your Turn 1

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A possible solution on the next page.

Example 2   **Write an Equation Involving a Parallel Line**

a) Write the equation of a line that is parallel to $2x - y + 4 = 0$
and through $(-1,6)$. Express the equation in slope-intercept form.

$$2x - y + 4 = 0$$
$$+y \qquad +y$$
$$\boxed{2}x + 4 = y$$

$$y - y_1 = 2(x - x_1)$$
$$y - 6 = 2(x - (-1))$$
$$y - 6 = 2(x + 1)$$
$$y - 6 = 2x + 2$$
$$+6 \qquad +6$$

b) Write the equation in general form. $\boxed{y = 2x + 8}$

$$0 = 2x - y + 8$$

c) Use technology to verify that the lines are parallel.

Example 2

Example 2   **Write an Equation Involving a Parallel Line**

a) Write the equation of a line that is parallel to $2x - y + 4 = 0$
and through $(-1,6)$. Express the equation in slope-intercept form.

$$2x - y + 4 = 0 \qquad\qquad y - y_1 = m(x - x_1)$$
$$-y = -2x - 4 \qquad\qquad y - 6 = 2(x - (-1))$$
$$y = 2x + 4 \qquad\qquad y - 6 = 2x + 2$$
$$m = 2 \qquad\qquad\qquad y = 2x + 8$$

b) Write the equation in general form.

$$y = 2x + 8$$
$$-2x + y - 8 = 0$$
$$2x - y + 8 = 0$$

c) Use technology to verify that the lines are parallel.

Example 2

Your Turn

Write the equation of a line that is parallel to $3x + y + 3 = 0$ and passes through $(5, -6)$. Express the equation in slope-intercept form and in general form. Use technology to verify that the lines are parallel.

$3x + y + 3 = 0$
$-3x \quad -3 \quad -3x - 3$

$y = -3x - 3$

$y - y_1 = m(x - x_1)$

$y - (-6) = -3(x - 5)$

$y + 6 = -3(x - 5)$

$y + 6 = -3x + 15$
$ -6 -6$

$\boxed{y = -3x + 9}$

$+3x - 9 \qquad +3x - 9$

$3x + y - 9 = 0$

Your Turn 2

Your Turn

Write the equation of a line that is parallel to $3x + y + 3 = 0$ and passes through $(5, -6)$. Express the equation in slope-intercept form and in general form. Use technology to verify that the lines are parallel.

$3x + y + 3 = 0$

$y = -3x - 3 \qquad\qquad (5, -6)$

$m = -3$

$y - y_1 = m(x - x_1)$

$y - (-6) = -3(x - 5)$

$y + 6 = -3x + 15$

$y = -3x + 9$

$y = -3x + 9$

$3x + y - 9 = 0$

Your Turn 2

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Example 3 Write an Equation Involving a Perpendicular Line

A possible solution on the next page.

Write the equation of a line perpendicular to $3x + 2y - 6 = 0$ with an x-intercept of 9. Express the equation in slope-intercept form and in general form.

$$3x + 2y - 6 = 0$$
$$-3x \qquad +6 \quad -3x + 6$$

$$\frac{2y}{2} = \frac{-3x}{2} + \frac{6}{2}$$

$$y = \boxed{-\frac{3}{2}}x + 3$$

$\frac{3}{2} \rightarrow$ Perpendicular

$\downarrow$

$\boxed{\frac{2}{3}}$

x-int of 9

(9,0)

$$y - y_1 = m(x - x_1)$$
$$y - 0 = \frac{2}{3}(x - 9)$$
$$y = \frac{2}{3}x - 6$$
$$-y \qquad -y$$
$$3\left[0 = \frac{2}{3}x - y - 6\right]$$
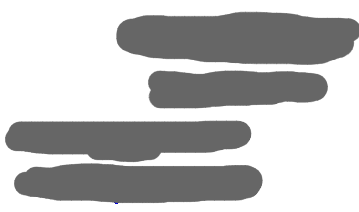$$0 = 2x - 3y - 18$$

Example 3

Example 3 Write an Equation Involving a Perpendicular Line

Write the equation of a line perpendicular to $3x + 2y - 6 = 0$ with an x-intercept of 9. Express the equation in slope-intercept form and in general form.

$$3x + 2y - 6 = 0$$
$$2y = -3x + 6$$
$$y = \frac{-3}{2}x + 3$$

x-intercept @ 9

(9,0)

$$m = \frac{-3}{2}$$
$$m_\perp = \frac{2}{3}$$

$$y - y_1 = m(x - x_1)$$
$$y - 0 = \frac{2}{3}(x - 9)$$

Example 3

Your Turn

A line is perpendicular to $4x + y - 12 = 0$ and passes through $(8, -6)$. Write the equation of the line in either slope-intercept form or general form.

Your Turn 3

Your Turn

A line is perpendicular to $4x + y - 12 = 0$ and passes through $(8, -6)$. Write the equation of the line in either slope-intercept form or general form.

$$4x + y - 12 = 0$$

$$y = -4x + 12$$

$$(8, -6)$$

$$m = -4$$

$$m_\perp = \tfrac{1}{4}$$

$$y - y_1 = m(x - x_1)$$

$$y - (-6) = \tfrac{1}{4}(x - 8)$$

$$y + 6 = \tfrac{1}{4}x - 2$$

$$y = \tfrac{1}{4}x - 8$$

$$4(y) = \left(\tfrac{1}{4}x - 8\right)4$$

$$4y = x - 32$$

$$-x + 4y + 32 = 0$$

$$x - 4y - 32 = 0$$

Your Turn 3

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Assignment:

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Challenge Questions: 19, 21, 23

Assig