8.3 Number of Solutions for Systems of Linear Equations

- Solve problems that involve systems of linear equations in two variables graphically

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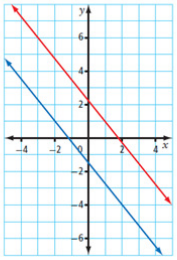
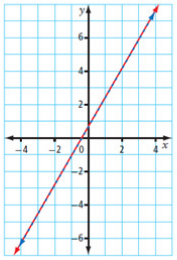
Parallel lines do not intersect at all. So, a system of parallel lines has no solution.

Coincident lines have an infinite number of solutions because the lines are equivalent. They overlap.

Parallel Lines



Coincident Lines



Reducing the equation to lowest terms may help you identify whether the equations are equivalent. If they are equivalent, then they must have an infinite number of solutions.

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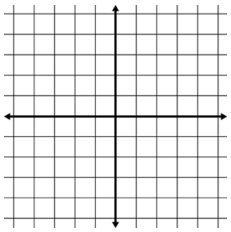
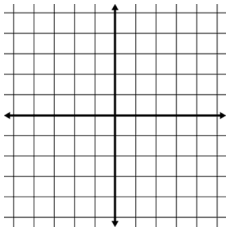
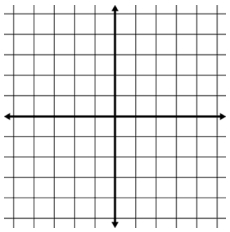
Example 1 Predict and Check the Number of Solutions

Predict the number of solutions for each system of linear equations. Explain your reasoning, and then check each answer by graphing the linear system.

a) $y = 2x - 3$
$y = \frac{1}{2}x + 3$

b) $4x + 10y = 30$
$2x + 5y = 35$

c) $10x - 6y = -12$
$21y = 42 + 35x$

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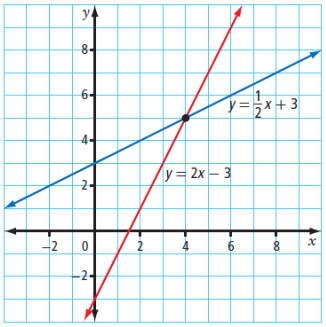
Solution

a) The slope of $y = 2x - 3$ is 2. The slope of $y = \frac{1}{2}x + 3$ is $\frac{1}{2}$.



The equations have different slopes. So, the graph will result in two lines that intersect at one point. Therefore, this system has one solution.

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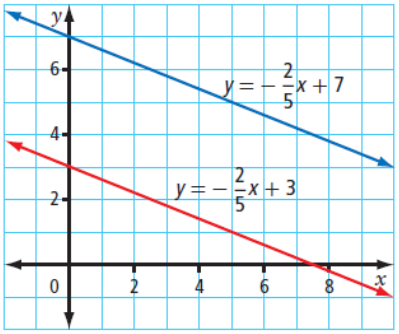
b) Rearrange each equation to slope-intercept form by isolating y.

$$\begin{aligned} 4x + 10y &= 30 \\ 4x + 10y - 4x &= 30 - 4x \\ 10y &= -4x + 30 \\ y &= \frac{-2}{5}x + 3 \end{aligned} \qquad \begin{aligned} 2x + 5y &= 35 \\ 2x + 5y - 2x &= 35 - 2x \\ 5y &= -2x + 35 \\ y &= \frac{-2}{5}x + 7 \end{aligned}$$

Since the lines have the same slope and different y-intercepts, the graph will result in parallel lines. The lines will never intersect. Therefore, this linear system has no solutions.



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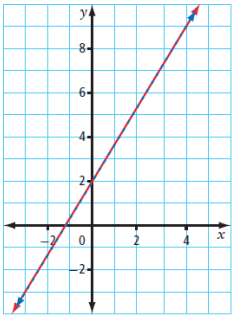
c) For the linear system $10x - 6y = -12$ and $21y = 42 + 35x$, isolate y in each equation to compare the slopes and y-intercepts.

$$\begin{aligned} 10x - 6y &= -12 \\ 10x - 6y + 6y + 12 &= -12 + 6y + 12 \\ 10x + 12 &= 6y \\ \frac{5}{3}x + 2 &= y \\ y &= \frac{5}{3}x + 2 \end{aligned}$$

$$\begin{aligned} 21y &= 42 + 35x \\ \frac{21y}{21} &= \frac{42}{21} + \frac{35x}{21} \\ y &= 2 + \frac{5}{3}x \\ y &= \frac{5}{3}x + 2 \end{aligned}$$

Both equations have a slope of $\frac{5}{3}$ and a y-intercept of 2.

The graph will result in coincident lines. Therefore, this linear system has an infinite number of solutions.



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Your Turn $y=mx+b$

Predict the number of solutions for each system of linear equations. Justify your answers.

a) $x+2y=4$
$y=-\frac{1}{2}x+4$

b) $6y-4x=6$
$y=\frac{2}{3}x+1$

c) $y=3x-1$
$y=2x-1$

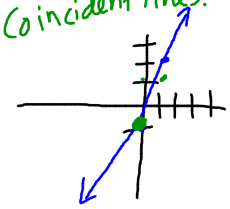
$x+2y=4$
$-x$
$\frac{2y}{2}=-\frac{x}{2}+\frac{4}{2}$
$y=-\frac{1}{2}x+2$

No solution
Slopes are the same
Parallel lines

$6y-4x=6$
$+4x \quad +4x$
$\frac{6y}{6}=\frac{4x}{6}+\frac{6}{6}$
$y=\frac{2}{3}x+1$

Different Slopes. 1 Solution

$y=\frac{2}{3}x+1$
Infinite number of solutions
Coincident lines.



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Example 3 Identify Zero and Infinite Solutions by Comparing Coefficients

Sabrina's teacher gives her the following systems of linear equations and tells her that each system has either no solution or an infinite number of solutions. How can Sabrina determine each answer by inspecting the equations?

a) $2x+3y=12$
$2x+3y=20$

b) $2x+3y=12$
$\frac{4x}{2}+\frac{6y}{2}=\frac{24}{2}$
$2x+3y=12$

Solution

a) L.S. ≠ R.S. for both equations
∴ No solution ∴ the lines are parallel

b) Equations are equal ∴ ← therefore Infinite number of solutions

Your turn p.11, 12

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Your Turn

Determine, by inspection, whether each linear system has an infinite number of solutions or no solution. Explain your reasoning.

a) $2x+10y-16=0$
$x+5y-8=0$

$\frac{2}{2}x+\frac{10}{2}y-\frac{16}{2}=\frac{0}{2}$

$x+5y-8=0$

Infinite # of solutions lines are coincident

b) $x+2y+4=0$
$x+2y-6=0$

The L.S. $\neq$ R.S. for both equations.

No solution lines are parallel

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